

General Instructions:

1. All questions are compulsory.
2. The question paper consists of **29** questions divided into three sections **A, B** and **C**. **Section A** comprises of **10** questions of **1** mark each, **Section B** comprises of **12** questions of **4** marks each and **Section C** comprises of **7** questions of **6** marks each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. Use of calculator is not permitted. You may ask for **logarithmic tables**, if required.

SECTION A

- 1) Explain with reason whether the following statement is true or false
If $x \in A$ and $A \in B$, then $x \in B$
- 2) Let $A = \emptyset$. Find the number of elements in $P(P(A))$ and list them.
- 3) Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find $A \times (B \cap C)$
- 4) Let $f(x) = 3x + 1$, $g(x) = \frac{|x| - 1}{3}$. Find $(f+g)(-2)$ and $\left(\frac{g}{f}\right)(2)$
- 5) Find the value of $\sin 765^\circ$
- 6) Given $\sin x = \frac{-3}{5}$, x lies in third quadrant, Find the value of $\tan x$.
- 7) A point P moves in such a way that its sum of the distance from two fixed is fixed. Identify the conic.
- 8) Sketch the conic $x^2 = -9y$, find the focus and the length of latus rectum.
- 9) Three events A, B and C are mutually exclusive and exhaustive; if $P(A) = 3/5$ and $P(B) = 1/6$, find $P(C)$.
- 10) If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x

SECTION B

- 11) For any set A and B , show that $P(A \cap B) = P(A) \cap P(B)$
- 12) Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Are the following true? i) $(a, b) \in R$, implies $(b, a) \in R$ ii) $(a, a) \in R$, for all $a \in N$ iii) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$
- 13) Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$
- 14) Find the general solution of $2\cos^2 x + 3\sin x = 0$

- 15) If $\left(\frac{1+i}{1-i}\right)^2 = 1$, then find the least positive integral value of m .
- 16) Find the equation of the line perpendicular to the line $x-7y+5=0$ and have x intercept 3.
- 17) Find the equation of the ellipse, with major axis along the x -axis and passing through the points (4,3) and (6,2).
- 18) A circle has radius 3 units and its centre lies on the line $y=x-1$. Find the equation of the circle if it passes through (7,3).
- 19) Find the coordinate of a point(s) on y -axis which are at a distance of $5\sqrt{2}$ from the point (3,-2,5).
- 20) Find the numbers of different 8-letter arrangements that, can be made from the letters of the word 'DAUGHTER' such that all vowels do not occur together.
- 21) A committee of two person is selected from two men and two women. What is the probability that the committee will have a) no man b) one man
- 22) The probability that at least one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, find $P(A') + P(B')$.

SECTION-C

- 23) Show by the Principle of Mathematical Induction that $x^{2n-1} + y^{2n-1}$ is divisible by $x+y$, for all $n \in \mathbf{N}$.
- 24) Given that $z_1 z_2 \neq 0$, prove that $\operatorname{Re}(z_1 \bar{z}_2) = |z_1| |z_2| \cos(\theta_1 - \theta_2)$ if and only if $\theta_1 - \theta_2 = 2n\pi$, $n \in \mathbf{Z}$, $\theta_1 = \operatorname{Arg}(z_1)$, $\theta_2 = \operatorname{Arg}(z_2)$
- 25) If p is the length of perpendicular from the origin to the line whose intercept on the axes are a and b , then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.
- 26) Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2=12y$ to the end points of the latus rectum.
- 27) a) How many 6-digit numbers can be formed from the digits 0,1,3,5,7 and 9 which are divisible by 10 and no digit is repeated?
b) Show that ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$.
- 28) m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated together is
$$\frac{m!(m+1)!}{(m-n+1)!}$$
- 29) If z_1 and z_2 are two complex number prove that $|z_1 + z_2| \leq |z_1| + |z_2|$

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