35761

MULTIPLE CHOICE QUESTIONS

SUBJECT : MATHEMATICS

FULL MARKS : 80

(Each question carries one mark)

(English Version)

1. Two dice are tossed once. The probability of getting an even number at the first die or a total of 8 is

   A. $\frac{1}{36}$ B. $\frac{3}{36}$ C. $\frac{11}{36}$ D. $\frac{23}{36}$

2. The probability that at least one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then $P(A')+P(B')$ is

   A. 0.9 B. 0.15 C. 1.1 D. 1.2

3. The value of $\frac{\log_3 5\times\log_{25}27\times\log_{49}7}{\log_{81}3}$ is

   A. 1 B. 6 C. $\frac{2}{3}$ D. 3

4. In a right-angled triangle, the sides are a, b and c, with c as hypotenuse, and $c-b\neq 1$, $c+b\neq 1$. Then the value of $(\log_{c+b}a+\log_{c-b}a)/(2\log_{c+b}a\times\log_{c-b}a)$ will be

   A. 2 B. –1 C. $\frac{1}{2}$ D. 1

5. Sum of n terms of the following series $1^3+3^3+5^3+7^3+\cdots\cdots$ is

   A. $n^2(2n^2-1)$ B. $n^3(n-1)$ C. n^3+8n+4 D. $2n^4+3n^2$

6. G. M. and H. M. of two numbers are 10 and 8 respectively. The numbers are :

   A. 5, 20 B. 4, 25 C. 2, 50 D. 1, 100

7. The value of n for which $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}$ is the geometric mean of x and y is

   A. $n=-\frac{1}{2}$ B. $n=\frac{1}{2}$ C. $n=1$ D. $n=-1$

8. If angles A, B and C are in A.P., then $\frac{a+c}{b}$ is equal to

   A. $2\sin\frac{A-C}{2}$ B. $2\cos\frac{A-C}{2}$ C. $\cos\frac{A-C}{2}$ D. $\sin\frac{A-C}{2}$

9. If $\frac{\cos A}{3}=\frac{\cos B}{4}=\frac{1}{5}$, $-\frac{\pi}{2}<A<0$, $-\frac{\pi}{2}<B<0$ then value of $2\sin A+4\sin B$ is

   A. 4 B. –2 C. -4 D. 0

10. The value of $\frac{\cot 54^\circ}{\tan 36^\circ}+\frac{\tan 20^\circ}{\cot 70^\circ}$ is

    A. 0 B. 2 C. 3 D. 1

11. If $\sin 6\theta+\sin 4\theta+\sin 2\theta=0$, then the general value of θ is

    A. $\frac{n\pi}{4}, n\pi\pm\frac{\pi}{3}$ B. $\frac{n\pi}{4}, n\pi\pm\frac{\pi}{6}$ C. $\frac{n\pi}{4}, 2n\pi\pm\frac{\pi}{3}$ D. $\frac{n\pi}{4}, 2n\pi\pm\frac{\pi}{6}$

12. In a ΔABC, $2ac\sin\frac{A-B+C}{2}$ is equal to

A. $a^2+b^2-c^2$ B. $c^2+a^2-b^2$ C. $b^2-a^2-c^2$ D. $c^2-a^2-b^2$

13. Value of $\tan^{-1}\left(\frac{\sin 2-1}{\cos 2}\right)$ is

A. $\frac{\pi}{2}-1$ B. $1-\frac{\pi}{4}$ C. $2-\frac{\pi}{2}$ D. $\frac{\pi}{4}-1$

14. The straight line $3x+y=9$ divides the line segment joining the points (1, 3) and (2, 7) in the ratio

A. 3 : 4 externally B. 3 : 4 internally C. 4 : 5 internally D. 5 : 6 externally

15. If the sum of distances from a point P on two mutually perpendicular straight lines is 1 unit, then the locus of P is

A. a parabola B. a circle C. an ellipse D. a straight line

16. The straight line $x+y-1=0$ meets the circle $x^2+y^2-6x-8y=0$ at A and B. Then the equation of the circle of which AB is a diameter is

A. $x^2+y^2-2y-6=0$ B. $x^2+y^2+2y-6=0$

C. $2(x^2+y^2)+2y-6=0$ D. $3(x^2+y^2)+2y-6=0$

17. If t_1 and t_2 be the parameters of the end points of a focal chord for the parabola $y^2=4ax$, then which one is true ?

A. $t_1t_2=1$ B. $\frac{t_1}{t_2}=1$ C. $t_1t_2=-1$ D. $t_1+t_2=-1$

18. S and T are the foci of an ellipse and B is an end point of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

19. For different values of α, the locus of the point of intersection of the two straight lines $\sqrt{3}x-y-4\sqrt{3}\alpha=0$ and $\sqrt{3}\alpha x+\alpha y-4\sqrt{3}=0$ is

A. a hyperbola with eccentricity 2

B. an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

C. a hyperbola with eccentricity $\sqrt{\frac{19}{16}}$

D. an ellipse with eccentricity $\frac{3}{4}$

20. The area of the region bounded by $y^2=x$ and $y=|x|$ is

A. $\frac{1}{3}$ sq. unit B. $\frac{1}{6}$ sq. unit C. $\frac{2}{3}$ sq. unit D. 1 sq. unit

21. If the displacement, velocity and acceleration of a particle at time t be x, v and f respectively, then which one is true ?

A. $f=v^3\frac{d^2t}{dx^2}$ B. $f=-v^3\frac{d^2t}{dx^2}$ C. $f=v^2\frac{d^2t}{dx^2}$ D. $f=-v^2\frac{d^2t}{dx^2}$

22. The displacement x of a particle at time t is given by $x=At^2+Bt+C$, where A, B, C are constants and v is velocity of a particle, then the value of $4Ax-v^2$ is

A. $4AC+B^2$ B. $4AC-B^2$ C. $2AC-B^2$ D. $2AC+B^2$

23. For what values of x, the function $f(x)=x^4-4x^3+4x^2+40$ is monotone decreasing ?

A. $0<x<1$ B. $1<x<2$ C. $2<x<3$ D. $4<x<5$

24. The displacement of a particle at time t is x, where $x=t^4-kt^3$. If the velocity of the particle at time $t=2$ is minimum, then

A. $k=4$ B. $k=-8$ C. $k=8$ D. $k=-8$

25. The point in the interval $[0,2\pi]$, where $f(x)=e^x\sin x$ has maximum slope, is

A. $\pi/4$ B. $\pi/2$ C. π D. $3\pi/2$

26. The minimum value of $f(x)=e^{(x^4-x^3+x^2)}$ is

A. e B. $-e$ C. 1 D. -1

27. $\int\frac{\log\sqrt{x}}{3x}dx$ is equal to

A. $\frac{1}{3}(\log\sqrt{x})^2+C$ B. $\frac{2}{3}(\log\sqrt{x})^2+C$ C. $\frac{2}{3}(\log x)^2+C$ D. $\frac{1}{3}(\log x)^2+C$

28. $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx$ is equal to

A. $\frac{e^x}{x}+C$ B. $\frac{e^x}{2x^2}+C$ C. $\frac{2e^x}{x}+C$ D. $\frac{2e^x}{x^2}+C$

29. The value of the integral $\int\frac{dx}{\left(e^x+e^{-x}\right)^2}$ is

A. $\frac{1}{2}\left(e^{2x}+1\right)+C$ B. $\frac{1}{2}\left(e^{-2x}+1\right)+C$ C. $-\frac{1}{2}\left(e^{2x}+1\right)^{-1}+C$ D. $\frac{1}{4}\left(e^{2x}-1\right)+C$

30. The value of $\lim_{x\to0}\frac{\sin^2x+\cos x-1}{x^2}$ is

A. 1 B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 0

31. The value of $\lim_{x\to0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$ is

A. e^2 B. e C. $\frac{1}{e}$ D. $\frac{1}{e^2}$

32. In which of the following functions, Rolle's theorem is applicable ?

A. $f(x)=|x|$ in $-2\le x\le2$ B. $f(x)=\tan x$ in $0\le x\le\pi$

C. $f(x)=1+(x-2)^{2/3}$ in $1\le x\le3$ D. $f(x)=x(x-2)^2$ in $0\le x\le2$

33. If $f(5)=7$ and $f'(5)=7$, then $\lim_{x\to5}\frac{xf(5)-5f(x)}{x-5}$ is given by

A. 35 B. -35 C. 28 D. -28

34. If $y=(1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{2n})$ then the value of $\left(\frac{dy}{dx}\right)_{x=0}$ is

A. 0 B. -1 C. 1 D. 2

35. The value of $f(0)$ so that the function $f(x)=\frac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{1}{6}$ D. $\frac{1}{8}$

36. $\int\sqrt{1+\cos x}\,dx$ is equal to

A. $2\sqrt{2}\cos\frac{x}{2}+C$ B. $2\sqrt{2}\sin\frac{x}{2}+C$ C. $\sqrt{2}\cos\frac{x}{2}+C$ D. $\sqrt{2}\sin\frac{x}{2}+C$

37. The function $f(x)=\sec\left[\log\left(x+\sqrt{1+x^2}\right)\right]$ is

A. odd B. even C. neither odd nor even D. constant

38. $\lim_{x\to 0}\frac{\sin|x|}{x}$ is equal to

A. 1 B. 0 C. positive infinity D. does not exist

39. The co-ordinates of the point on the curve $y=x^2-3x+2$ where the tangent is perpendicular to the straight line $y=x$ are

A. (0, 2) B. (1, 0) C. $(-1, 6)$ D. (2, –2)

40. The domain of the function $f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$ is

A. $(-3, 3)$ B. $[-3, 3]$ C. $(-\infty,-3)\cup(3,\infty)$ D. $(-\infty,-3]\cup[3,\infty)$

41. If the line $ax+by+c=0$ is a tangent to the curve $xy=4$, then

A. $a<0,\ b>0$ B. $a\le 0,\ b>0$ C. $a<0,\ b<0$ D. $a\le 0,\ b<0$

42. If the normal to the curve $y=f(x)$ at the point (3, 4) makes an angle $3\pi/4$ with the positive x-axis, then $f'(3)$ is

A. 1 B. -1 C. $-\frac{3}{4}$ D. $\frac{3}{4}$

43. The general solution of the differential equation $100\frac{d^2y}{dx^2}-20\frac{dy}{dx}+y=0$ is

A. $y=(c_1+c_2x)e^x$ B. $y=(c_1+c_2x)e^{-x}$ C. $y=(c_1+c_2x)e^{\frac{x}{10}}$ D. $y=c_1e^x+c_2e^{-x}$

44. If $y''-3y'+2y=0$ where $y(0)=1$, $y'(0)=0$, then the value of y at $x=\log_e 2$ is

A. 1 B. -1 C. 2 D. 0

45. The degree of the differential equation $x=1+\left(\frac{dy}{dx}\right)+\frac{1}{2!}\left(\frac{dy}{dx}\right)^2+\frac{1}{3!}\left(\frac{dy}{dx}\right)^3+\ldots\ldots$

A. 3 B. 2 C. 1 D. not defined

46. The equation of one of the curves whose slope at any point is equal to $y+2x$ is

A. $y=2(e^x+x-1)$ B. $y=2(e^x-x-1)$ C. $y=2(e^x-x+1)$ D. $y=2(e^x+x+1)$

47. Solution of the differential equation $xdy - ydx = 0$ represents a

A. parabola B. circle C. hyperbola D. straight line

48. The value of the integral $\int_0^{\pi/2} \sin^5 x\,dx$ is

A. $\frac{4}{15}$ B. $\frac{8}{5}$ C. $\frac{8}{15}$ D. $\frac{4}{5}$

49. If $\frac{d}{dx}\{f(x)\} = g(x)$, then $\int_a^b f(x)g(x)\,dx$ is equal to

A. $\frac{1}{2}\left[f^2(b) - f^2(a)\right]$ B. $\frac{1}{2}\left[g^2(b) - g^2(a)\right]$ C. $f(b) - f(a)$ D. $\frac{1}{2}\left[f(b^2) - f(a^2)\right]$

50. If $I_1 = \int_0^{3\pi} f(\cos^2 x)\,dx$ and $I_2 = \int_0^{\pi} f(\cos^2 x)\,dx$, then

A. $I_1 = I_2$ B. $3I_1 = I_2$ C. $I_1 = 3I_2$ D. $I_1 = 5I_2$

51. The value of $I = \int_{-\pi/2}^{\pi/2} |\sin x|\,dx$ is

A. 0 B. 2 C. –2 D. $-2 < I < 2$

52. If $I = \int_0^1 \frac{dx}{1 + x^{\pi/2}}$, then

A. $\log_e 2 < I < \pi/4$ B. $\log_e 2 > I$ C. $I = \pi/4$ D. $I = \log_e 2$

53. The area enclosed by, $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

A. 12 sq. units B. 13 sq. units C. $13\frac{1}{2}$ sq. units D. 14 sq. units

54. The area bounded by the parabolas $y = 4x^2$, $y = \frac{x^2}{9}$ and the line $y = 2$ is

A. $\frac{5\sqrt{2}}{3}$ sq. units B. $\frac{10\sqrt{2}}{3}$ sq. units C. $\frac{15\sqrt{2}}{3}$ sq. units D. $\frac{20\sqrt{2}}{3}$ sq. units

55. The equation of normal of $x^2 + y^2 - 2x + 4y - 5 = 0$ at (2, 1) is

A. $y = 3x - 5$ B. $2y = 3x - 4$ C. $y = 3x + 4$ D. $y = x + 1$

56. If the three points $(3q, 0)$, $(0, 3p)$ and (1, 1) are collinear then which one is true ?

A. $\frac{1}{p} + \frac{1}{q} = 1$ B. $\frac{1}{p} + \frac{1}{q} = 1$ C. $\frac{1}{p} + \frac{1}{q} = 3$ D. $\frac{1}{p} + \frac{3}{q} = 1$

57. The equations $y = \pm\sqrt{3}x$, $y = 1$ are the sides of

A. an equilateral triangle B. a right angled triangle
C. an isosceles triangle D. an obtuse angled triangle

58. The equations of the lines through (1, 1) and making angles of 45° with the line $x + y = 0$ are

A. $x - 1 = 0$, $x - y = 0$ B. $x - y = 0$, $y - 1 = 0$
C. $x + y - 2 = 0$, $y - 1 = 0$ D. $x - 1 = 0$, $y - 1 = 0$

59. In a triangle PQR, $\angle R = \pi/2$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of $ax^2+bx+c=0$, where $a \neq 0$, then which one is true ?

A. $c = a+b$ B. $a = b+c$ C. $b = a+c$ D. $b = c$

60. The value of $\dfrac{\sin 55^\circ - \cos 55^\circ}{\sin 10^\circ}$ is

A. $\frac{1}{\sqrt{2}}$ B. 2 C. 1 D. $\sqrt{2}$

61. The value of $\dfrac{\cot x - \tan x}{\cot 2x}$ is

A. 1 B. 2 C. –1 D. 4

62. The number of points of intersection of $2y = 1$ and $y = \sin x$, in $-2\pi \leq x \leq 2\pi$ is

A. 1 B. 2 C. 3 D. 4

63. Let R be the set of real numbers and the mapping $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 5 - x^2$ and $g(x) = 3x - 4$, then the value of $(fog)(-1)$ is

A. –44 B. –54 C. –32 D. –64

64. If $A = \{1,2,3,4\}$, $B = \{1,2,3,4,5,6\}$ are two sets, and function $f : A \to B$ is defined by $f(x) = x+2 \ \forall x \in A$, then the function f is

A. bijective B. onto C. one–one D. many–one

65. If the matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$, then AB will be

A. $\begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$ B. $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ C. $\begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

66. If ω is an imaginary cube root of unity and $\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$ then one of the values of x is

A. 1 B. 0 C. –1 D. 2

67. If $A = \begin{bmatrix} 1 & 2 \\ -4 & -1 \end{bmatrix}$ then A^{-1} is

A. $\frac{1}{7}\begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$ B. $\frac{1}{7}\begin{bmatrix} 1 & 2 \\ -4 & -1 \end{bmatrix}$ C. $\frac{1}{7}\begin{bmatrix} -1 & -2 \\ 4 & 1 \end{bmatrix}$ D. Does not exist

68. The value of $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots\cdots$ is

A. $e^{\frac{1}{2}}$ B. e^{-1} C. e D. $e^{-\frac{1}{3}}$

69. If sum of an infinite geometric series is $\frac{4}{3}$ and its 1st term is $\frac{3}{4}$, then its common ratio is

A. $\frac{7}{16}$ B. $\frac{9}{16}$ C. $\frac{1}{9}$ D. $\frac{7}{9}$

70. The number of permutations by taking all letters and keeping the vowels of the word COMBINE in the odd places is

A. 96 B. 144 C. 512 D. 576

71. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$, then n is just greater than integer

A. 5 B. 6 C. 4 D. 7

72. If in the expansion of $(a-2b)^n$, the sum of the 5th and 6th term is zero, then the value of $\frac{a}{b}$ is

A. $\frac{n-4}{5}$ B. $\frac{2(n-4)}{5}$ C. $\frac{5}{n-4}$ D. $\frac{5}{2(n-4)}$

73. $\left(2^{3n}-1\right)$ will be divisible by $(\forall n \in N)$

A. 25 B. 8 C. 7 D. 3

74. Sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of x is

A. 2^{59} B. 2^{58} C. 2^{30} D. 2^{29}

75. If $\left(1-x+x^2\right)^n = a_0 + a_1x + a_2x^2 + \cdots\cdots + a_{2n}x^{2n}$, then the value of $a_0 + a_2 + a_4 + \cdots + a_{2n}$ is

A. $3^n + \frac{1}{2}$ B. $3^n - \frac{1}{2}$ C. $\frac{3^n-1}{2}$ D. $\frac{3^n+1}{2}$

76. If α, β be the roots of the quadratic equation $x^2 + x + 1 = 0$ then the equation whose roots are α^{19}, β^7 is

A. $x^2 - x + 1 = 0$ B. $x^2 - x - 1 = 0$ C. $x^2 + x - 1 = 0$ D. $x^2 + x + 1 = 0$

77. The roots of the quadratic equation $x^2 - 2\sqrt{3}x - 22 = 0$ are :

A. imaginary
B. real, rational and equal
C. real, irrational and unequal
D. real, rational and unequal

78. The quadratic equation $x^2 + 15|x| + 14 = 0$ has

A. only positive solutions
B. only negative solutions
C. no solution
D. both positive and negative solution

79. If $z = \frac{4}{1-i}$, then $\bar{z}$ is (where $\bar{z}$ is complex conjugate of z)

A. $2(1+i)$ B. $(1+i)$ C. $\frac{2}{1-i}$ D. $\frac{4}{1+i}$

80. If $-\pi < \arg(z) < -\frac{\pi}{2}$ then $\arg\bar{z} - \arg(-\bar{z})$ is

A. π B. $-\pi$ C. $\frac{\pi}{2}$ D. $-\frac{\pi}{2}$

[Turn Over for Bengali Version]

DESCRIPTIVE TYPE QUESTIONS

SUBJECT : MATHEMATICS

FULL MARKS : 20

(Each question carries two marks)

(English Version)

1. Prove that the equation $\cos 2x + a \sin x = 2a - 7$ possesses a solution if $2 \le a \le 6$.

2. Find the values of x, $(-\pi < x < \pi,\ x \neq 0)$ satisfying the equation, $8^{1+|\cos x|+|\cos^2 x|+\cdots\cdots\infty} = 4^3$

3. Prove that the centre of the smallest circle passing through origin and whose centre lies on $y = x+1$ is $\left(-\frac{1}{2}, \frac{1}{2}\right)$

4. Prove by induction that for all $n \in N$, $n^2 + n$ is an even integer $(n \ge 1)$.

5. If A, B are two square matrices such that AB = A and BA = B, then prove that $B^2 = B$

6. If $N = n!$ $(n \in N,\ n > 2)$, then find $\lim_{N \to \infty} \left[(\log_2 N)^{-1} + (\log_3 N)^{-1} + \cdots\cdots + (\log_n N)^{-1} \right]$

7. Use the formula $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$, to compute $\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$.

8. If $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$, prove that, $x\sqrt{1-y^2} + y\sqrt{1-x^2} = A$ where A is a constant.

9. Evaluate the following integral $\int_{-1}^{2} |x \sin \pi x| \, dx$

10. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$ and $g'(a) = 2$, find the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$.

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