

435261

10. The modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$ is
- A. $\sqrt{5}$ unit B. $\frac{\sqrt{11}}{5}$ unit C. $\frac{\sqrt{5}}{5}$ unit D. $\frac{\sqrt{12}}{5}$ unit
11. The equation of the tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at $(2, 1)$ is
- A. $x + 2 = 0$ B. $2x + 1 = 0$ C. $x + y + 1 = 0$ D. $x - 2 = 0$
12. A and B are two independent events such that $P(A \cup B') = 0.8$, and $P(A) = 0.3$. Then $P(B)$ is
- A. $\frac{2}{7}$ B. $\frac{2}{3}$ C. $\frac{3}{8}$ D. $\frac{1}{8}$
13. The total number of tangents through the point $(3, 5)$ that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ is
- A. 0 B. 2 C. 3 D. 4
14. The value of $\lim_{n \rightarrow \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2} \right]$ is
- A. $\frac{\pi}{4}$ B. $\log 2$ C. 0 D. 1
15. A particle is moving in a straight line. At time t , the distance between the particle from its starting point is given by $x = t - 6t^2 + t^3$. Its acceleration will be zero at
- A. $t = 1$ unit time B. $t = 2$ unit time C. $t = 3$ unit time D. $t = 4$ unit time
16. Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is
- A. $\frac{1}{190}$ B. $\frac{1}{120}$ C. $\frac{3}{190}$ D. $\frac{5}{190}$
17. The co-ordinates of the foot of the perpendicular from $(0, 0)$ upon the line $x + y = 2$ are
- A. $(2, -1)$ B. $(-2, 1)$ C. $(1, 1)$ D. $(1, 2)$
18. If A is a square matrix then,
- A. $A + A^T$ is symmetric B. AA^T is skew-symmetric
C. $A^T + A$ is skew-symmetric D. A^TA is skew-symmetric
19. The equation of the chord of the circle $x^2 + y^2 - 4x = 0$ whose mid point is $(1, 0)$ is
- A. $y = 2$ B. $y = 1$ C. $x = 2$ D. $x = 1$
20. If $A^2 - A + I = 0$, then the inverse of the matrix A is
- A. $A - I$ B. $A - A$ C. $A + I$ D. A
21. If C is the reflection of $A(2, 4)$ in x -axis and B is the reflection of C in y -axis, then $|AB|$ is
- A. 20 B. $2\sqrt{5}$ C. $4\sqrt{5}$ D. 4
22. The value of $\cos 15^\circ \cos 7\frac{1}{2}^\circ \sin 7\frac{1}{2}^\circ$ is
- A. $\frac{1}{2}$ B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{16}$

3. The value of integral $\int_{-1}^1 \frac{|x+2|}{x+2} dx$ is
 A. 1 B. 2 C. 0 D. -1
4. The line $y = 2t^2$ intersects the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ in real points if
 A. $|t| \leq 1$ B. $|t| < 1$ C. $|t| > 1$ D. $|t| \geq 1$
5. General solution of $\sin x + \cos x = \min_{a \in \mathbb{R}} \{1, a^2 - 4a + 6\}$ is
 A. $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$ B. $2n\pi + (-1)^n \frac{\pi}{4}$ C. $n\pi + (-1)^{n+1} \frac{\pi}{4}$ D. $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$
6. If A and B are square matrices of the same order and $AB = 3I$, then A^{-1} is equal to
 A. $3B$ B. $\frac{1}{3}B$ C. $3B^{-1}$ D. $\frac{1}{3}B^{-1}$
7. The co-ordinates of the focus of the parabola described parametrically by $x = 5t^2 + 2$, $y = 10t + 4$ are
 A. (7, 4) B. (3, 4) C. (3, -4) D. (-7, 4)
8. For any two sets A and B, $A - (A - B)$ equals
 A. B B. A - B C. $A \cap B$ D. $A^c \cap B^c$
9. If $a = 2\sqrt{2}$, $b = 6$, $A = 45^\circ$, then
 A. no triangle is possible
 B. one triangle is possible
 C. two triangles are possible
 D. either no triangle or two triangles are possible
10. A Mapping from \mathbb{N} to \mathbb{N} is defined as follows :
 $f: \mathbb{N} \rightarrow \mathbb{N}$
 $f(n) = (n+5)^2$, $n \in \mathbb{N}$
 (\mathbb{N} is the set of natural numbers). Then
 A. f is not one-to-one
 B. f is onto
 C. f is both one-to-one and onto
 D. f is one-to-one but not onto
11. In a triangle ABC if $\sin A \sin B = \frac{ab}{c^2}$, then the triangle is
 A. equilateral B. isosceles C. right angled D. obtuse angled
12. $\int \frac{dx}{\sin x + \sqrt{3} \cos x}$ equals
 A. $\frac{1}{2} \ln |\tan(\frac{x}{2} - \frac{\pi}{6})| + c$ B. $\frac{1}{2} \ln |\tan(\frac{x}{4} - \frac{\pi}{6})| + c$ C. $\frac{1}{2} \ln |\tan(\frac{x}{2} + \frac{\pi}{6})| + c$ D. $\frac{1}{2} \ln |\tan(\frac{x}{4} + \frac{\pi}{3})| + c$

where c is an arbitrary constant.

$$A - I + A^{-1} = 0$$

5

$$\boxed{1} + \boxed{1} = \boxed{2}$$

$$A - A + B$$

435261

33. The value of $(1 + \cos \frac{\pi}{6})(1 + \cos \frac{\pi}{3})(1 + \cos \frac{2\pi}{3})(1 + \cos \frac{7\pi}{6})$ is
 A. $\frac{3}{16}$ B. $\frac{3}{8}$ C. $\frac{3}{4}$ D. $\frac{1}{2}$

34. If $P = \frac{1}{2} \sin^2 \theta + \frac{1}{3} \cos^2 \theta$ then
 A. $\frac{1}{3} \leq P \leq \frac{1}{2}$ B. $P \geq \frac{1}{2}$ C. $2 \leq P \leq 3$ D. $-\frac{\sqrt{13}}{6} \leq P \leq \frac{\sqrt{13}}{6}$

35. A positive acute angle is divided into two parts whose tangents are $\frac{1}{2}$ and $\frac{1}{3}$. Then the angle is
 A. $\frac{\pi}{4}$ B. $\frac{\pi}{5}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{6}$

36. If $f(x) = f(a - x)$ then $\int_0^a x f(x) dx$ is equal to
 A. $\int_0^a f(x) dx$ B. $\frac{a^2}{2} \int_0^a f(x) dx$ C. $\frac{a}{2} \int_0^a f(x) dx$ D. $-\frac{a}{2} \int_0^a f(x) dx$

37. The value of $\int_0^\infty \frac{dx}{(x^2 + 4)(x^2 + 9)}$ is
 A. $\frac{\pi}{60}$ B. $\frac{\pi}{20}$ C. $\frac{\pi}{40}$ D. $\frac{\pi}{80}$

38. If $I_1 = \int_0^{\frac{\pi}{4}} \sin^2 x dx$ and $I_2 = \int_0^{\frac{\pi}{4}} \cos^2 x dx$, then,
 A. $I_1 = I_2$ B. $I_1 < I_2$ C. $I_1 > I_2$ D. $I_2 = I_1 + \frac{\pi}{4}$

39. The second order derivative of $a \sin^3 t$ with respect to $a \cos^3 t$ at $t = \frac{\pi}{4}$ is

- A. 2 B. $\frac{1}{12a}$ C. $\frac{4\sqrt{2}}{3a}$ D. $\frac{3a}{4\sqrt{2}}$

40. The smallest value of $5 \cos \theta + 12$ is
 A. 5 B. 12

41. The general solution of the differential equation $\frac{dy}{dx} = e^{y+x} + e^{y-x}$ is
 A. $e^{-y} = e^x - e^{-x} + c$
 C. $e^{-y} = e^x + e^{-x} + c$

where c is an arbitrary constant

42. Product of any r consecutive natural numbers is always divisible by
 A. ! B. $(r+4)!$ C. $(r+1)!$ D. $(r+2)!$

43. The integrating factor of the differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$ is given by
 A. e^x B. $\log x$ C. $\log(\log x)$ D. x

- If $x^2 + y^2 = 1$ then
- $yy'' - (2y')^2 + 1 = 0$
 - $yy'' + (y')^2 + 1 = 0$
 - $yy'' - (y')^2 - 1 = 0$
 - $yy'' + 2(y')^2 + 1 = 0$
- If $c_0, c_1, c_2, \dots, c_n$ denote the co-efficients in the expansion of $(1+x)^n$ then the value of $c_1 + 2c_2 + 3c_3 + \dots + nc_n$ is
- $n \cdot 2^{n-1}$
 - $(n+1) \cdot 2^{n-1}$
 - $(n+1) \cdot 2^n$
 - $(n+2) \cdot 2^{n-1}$
- A polygon has 44 diagonals. The number of its sides is
- 10
 - 11
 - 12
 - 13
- If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is
- $\frac{4}{a+b}$
 - $\frac{1}{a+b}$
 - 0
 - 1
- The angle between the lines joining the foci of an ellipse to one particular extremity of the minor axis is 90° . The eccentricity of the ellipse is
- $\frac{1}{8}$
 - $\frac{1}{\sqrt{3}}$
 - $\frac{\sqrt{2}}{3}$
 - $\frac{1}{\sqrt{2}}$
- The order of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is
- 3
 - 2
 - 1
 - 4
- The sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is
- 7
 - 4
 - 1
 - 5
- If $\int_{-1}^4 f(x) dx = 4$ and $\int_2^4 \{3 - f(x)\} dx = 7$ then the value of $\int_{-1}^2 f(x) dx$ is
- 2
 - 3
 - 4
 - 5
- For each $n \in \mathbb{N}$, $2^{3n} - 1$ is divisible by
- 7
 - 8
 - 6
 - 16
- where \mathbb{N} is a set of natural numbers.
- The Rolle's theorem is applicable in the interval $-1 \leq x \leq 1$ for the function
- $f(x) = x$
 - $f(x) = x^2$
 - $f(x) = 2x^3 + 3$
 - $f(x) = |x|$
- The distance covered by a particle in t seconds is given by $x = 3 + 8t - 4t^2$. After 1 second its velocity will be
- 0 unit/second
 - 3 units/second
 - 4 units/second
 - 7 units/second
- If the co-efficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then the value of 'a' is
- $\frac{3}{7}$
 - $\frac{7}{3}$
 - $\frac{7}{9}$
 - $\frac{9}{7}$
- The value of $\left(\frac{1}{\log_3 12} + \frac{1}{\log_4 12} \right)$ is
- 0
 - $\frac{1}{2}$
 - 1
 - 2

MULTIPLE CHOICE QUESTIONS

SUBJECT - MATHEMATICS

FULL MARKS : 80

(Each question carries one mark)

(English Version)

Let $f(x) = \frac{\sqrt{x+3}}{x+1}$ then the value of $\lim_{x \rightarrow -3^+} f(x)$ is

- A. 0 B. does not exist C. $\frac{1}{2}$ D. $-\frac{1}{2}$

$f(x) = x + |x|$ is continuous for

- A. $x \in (-\infty, \infty)$ B. $x \in (-\infty, \infty) - \{0\}$ C. only $x > 0$ D. no value of x

$\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$ is equal to

- A. $\frac{2a}{b}$ B. $\frac{2b}{a}$ C. a/b D. b/a

If $i = \sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to

- A. 1 B. i C. i^n D. 0

5. $\int \frac{dx}{x(x+1)}$ equals

- A. $\ln\left|\frac{x+1}{x}\right| + c$ B. $\ln\left|\frac{x}{x+1}\right| + c$ C. $\ln\left|\frac{x-1}{x}\right| + c$ D. $\ln\left|\frac{(x-1)(x+1)}{x}\right| + c$

where c is an arbitrary constant.

6. If a, b, c are in G.P. ($a > 1, b > 1, c > 1$), then for any real number x (with $x > 0, x \neq 1$), $\log_a x, \log_b x, \log_c x$ are in

- A. G.P. B. A.P. C. H.P. D. G.P. but not in H.P.

7. A line through the point A (2, 0) which makes an angle of 30° with the positive direction of x -axis is rotated about

A in clockwise direction through an angle 15° . Then the equation of the straight line in the new position is

- A. $(2 - \sqrt{3})x + y - 4 + 2\sqrt{3} = 0$ B. $(2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$
 C. $(2 - \sqrt{3})x - y + 4 + 2\sqrt{3} = 0$ D. $(2 - \sqrt{3})x + y + 4 + 2\sqrt{3} = 0$

8. The equation $\sqrt{3} \sin x + \cos x = 4$ has

- A. only one solution B. two solutions C. infinitely many solutions D. no solution

9. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1, 1). The equation of the curve is

- A. $y = x^3 + 2$ B. $y = -x^3 - 2$ C. $y = 3x^3 + 4$ D. $y = -x^3 + 2$

57. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then the value of $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ will be
 A. $x + y + z$ B. 1 C. $ab + bc + ca$ D. \cancel{abc}
58. Using binomial theorem, the value of $(0.999)^3$ correct to 3 decimal places is
 A. 0.999 B. 0.998 C. 0.997 D. 0.995
59. If the rate of increase of the radius of a circle is 5 cm/sec., then the rate of increase of its area, when the radius is 20 cm, will be
 A. 10π B. 20π C. 200π D. 400π
60. The quadratic equation whose roots are three times the roots of $3ax^2 + 3bx + c = 0$ is
~~A.~~ $\cancel{ax^2 + 3bx + 3c = 0}$ B. $ax^2 + 3bx + c = 0$ C. $9ax^2 + 9bx + c = 0$ D. $ax^2 + bx + 3c = 0$
61. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
 A. $2\tan^{-1}\left(\frac{3}{4}\right)$ B. $\tan^{-1}\left(\frac{4}{3}\right)$ C. $\frac{\pi}{2}$ D. $\frac{\pi}{4}$
62. In triangle ABC, $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then B is equal to
 A. 30° B. 60° C. 90° D. 120°
63. $\int_0^{1000} e^{x-[x]}$ is equal to
 A. $\frac{e^{1000}-1}{e-1}$ B. $\frac{e^{1000}-1}{1000}$ C. $\frac{e-1}{1000}$ D. $1000(e-1)$
64. The coefficient of x^n , where n is any positive integer, in the expansion of $(1+2x+3x^2+\dots)^{\frac{1}{2}}$ is
~~A.~~ $\cancel{1}$ B. $\frac{n+1}{2}$ C. $2n+1$ D. $n+1$
65. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = a^2$ intersect at two distinct points if
 A. $a < 2$ B. $2 \leq a < 8$ C. $a > 8$ D. $a = 2$
66. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$ is equal to
 A. $\log(\sin^{-1}x) + c$ B. ~~C.~~ $\frac{1}{2}(\sin^{-1}x)^2 + c$ C. $\log\left(\sqrt{1-x^2}\right) + c$ D. $\sin(\cos^{-1}x) + c$
- where c is an arbitrary constant
67. The number of points on the line $x + y = 4$ which are unit distance apart from the line $2x + 2y = 5$ is
 A. 0 B. 1 C. 2 D. infinity
68. Simplest form of $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ is
 A. $\sec \frac{x}{2}$ B. $\sec x$ C. $\operatorname{cosec} x$

435261

If $y = \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

- A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. 1

If three positive real numbers a, b, c are in A.P. and $abc = 4$ then the minimum possible

- A. $2^{\frac{1}{2}}$ B. $2^{\frac{2}{3}}$ C. $2^{\frac{1}{3}}$ D. 2

If $5\cos 2\theta + 2\cos^2 \theta + 1 = 0$, when $(0 < \theta < \pi)$, then the values of θ are :

- A. $\frac{\pi}{3} \pm \pi$ B. $\frac{\pi}{3}, \cos^{-1}(\frac{3}{5})$ C. $\cos^{-1}(\frac{3}{5}) \pm \pi$ D. $\frac{\pi}{3}, \pi - \cos^{-1}(\frac{3}{5})$

For any complex number z , the minimum value of $|z| + |z - 1|$ is

- A. 0 B. 1 C. 2 D. -1

For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$ there is /are

- A. one pair of common tangents B. only one common tangent
C. three common tangents D. no common tangent

4. If C is a point on the line segment joining A (-3, 4) and B (2, 1) such that $AC = 2BC$, then the coordinate of C is

- A. $(\frac{1}{3}, 2)$ B. $(2, \frac{1}{3})$ C. (2, 7) D. (7, 2)

5. If a, b, c are real, then both the roots of the equation $(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$ are always

- A. positive B. negative C. real D. imaginary

6. The sum of the infinite series $1 + \frac{1}{2!} + \frac{1.3}{4!} + \frac{1.3.5}{6!} + \dots$ is

- A. e B. e^2 C. \sqrt{e} D. $\frac{1}{e}$

7. The point (-4, 5) is the vertex of a square and one of its diagonals is $7x - y + 8 = 0$. The equation of the other diagonal is

- A. $7x - y + 23 = 0$ B. $7y + x = 30$ C. $7y + x = 31$ D. $x - 7y = 30$

8. The domain of definition of the function $f(x) = \sqrt{1 + \log_e(1-x)}$ is

- A. $-\infty < x \leq 0$ B. $-\infty < x \leq \frac{e-1}{e}$ C. $-\infty < x \leq 1$

9. For what value of m, $\frac{a^{m+1} + b^{m+1}}{a^m + b^m}$ is the arithmetic mean of 'a' and 'b'?

- A. 1 B. 0 C. 2

10. The value of the limit $\lim_{x \rightarrow 1} \frac{\sin(e^{x-1} - 1)}{\log x}$ is

- A. 0 B. e C. $\frac{1}{e}$

$3d + 3B = -\frac{16}{a}$

$9d + 3B = \frac{3ac}{za}$

$$22x^2 + \frac{36x + 3c}{a} = 0$$

$$x^2 - \left(-\frac{36}{a}\right)x^2 + \frac{3c}{a}$$

[Turn Over for Bengali Version]

DESCRIPTIVE TYPE QUESTIONS

SUBJECT : MATHEMATICS

Full Marks - 20

(Each question carries two marks)

(English Version)

1. A train moving with constant acceleration takes t seconds to pass a certain fixed point and the front and back end of the train pass the fixed point with velocities u and v respectively. Show that the length of the train is $\frac{1}{2}(u + v)t$.

2. Show that

$$\frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}(\tan 27\theta - \tan\theta)$$

3. If $x = \sin t$, $y = \sin 2t$, prove that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$$

4. Show that, for a positive integer n , the coefficient of x^k ($0 \leq k \leq n$) in the expansion of

$$1 + (1+x) + (1+x)^2 + \dots + (1+x)^n \text{ is } {}^{n+1}C_{n-k}.$$

5. If m, n be integers, then find the value of $\int_{-\pi}^{\pi} (\cos mx - \sin nx)^2 dx$.

6. Find the angle subtended by the double ordinate of length $2a$ of the parabola $y^2 = ax$ at its vertex.

7. If f is differentiable at $x = a$, find the value of

$$\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$$

8. Find the values of ' a ' for which the expression $x^2 - (3a - 1)x + 2a^2 + 2a - 11$ is always positive.

9. Find the sum of the first n terms of the series $0.2 + 0.22 + 0.222 + \dots$

10. The equations to the pairs of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. Find the equations of its diagonals.

[Turn over for Bengali Version]