Trigonometrical Identities for Class IX and X

1. Prove that $\sin A\cot A+\sin A\operatorname{cosec}A=1+\cos A$. Soln:

$\sin A\cot A+\sin A\operatorname{cosec}A$

$=\sin A\dfrac{\cos A}{\sin A}+\sin A\dfrac{1}{\sin A}$

$=\cos A+1=R.H.S$

2. Prove that $\cot^2 A-\dfrac{1}{\sin^2 A}+1=0$. Soln:

$\cot^2 A-\dfrac{1}{\sin^2 A}+1$

$=\cot^2 A-\operatorname{cosec}^2 A+1$

$=-1+1$

$=0$

3. Prove that $\sec A(1-\sin A)(\sec A+\tan A)=1$. Soln:

$\sec A(1-\sin A)(\sec A+\tan A)$

$=\sec A(1-\sin A)\sec A(1+\sin A)$

$=\sec^2 A(1-\sin^2 A)$

$=\sec^2 A\cos^2 A$

$=1$

4. Show that $\dfrac{1-\tan^2 A}{\cot^2 A-1}=\tan^2 A$. Soln:

$\dfrac{1-\tan^2 A}{\cot^2 A-1}=\dfrac{1-\tan^2 A}{\dfrac{1}{\tan^2 A}-1}$

$=\dfrac{1-\tan^2 A}{\dfrac{1-\tan^2 A}{\tan^2 A}}=\tan^2 A$

5. Show that $\dfrac{\sin A}{1+\cos A}=\operatorname{cosec}A-\cot A$. Soln:

$\dfrac{\sin A}{1+\cos A}=\dfrac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$=\dfrac{\sin A(1-\cos A)}{(1-\cos^2 A)}=\dfrac{\sin A(1-\cos A)}{\sin^2 A}$

$=\dfrac{\sin A}{\sin^2 A}-\dfrac{\sin A\cos A}{\sin^2 A}=\operatorname{cosec}A-\cot A$

6. Show that $\dfrac{\sec A-1}{\sec A+1}=\dfrac{\cos A-1}{\cos A+1}$. Soln:

$\dfrac{\sec A-1}{\sec A+1}=\dfrac{\dfrac{1}{\cos A}-1}{\dfrac{1}{\cos A}+1}=\dfrac{\dfrac{1-\cos A}{\cos A}}{\dfrac{1+\cos A}{\cos A}}=\dfrac{1-\cos A}{1+\cos A}$

7. Show that $(1+\tan A)^2+(1-\tan A)^2=2\sec^2 A$. Soln:

$(1+\tan A)^2+(1-\tan A)^2=2(1^2+\tan^2 A)$

$=2\sec^2 A\left[(a-b)^2+(a+b)^2=2(a^2+b^2)\right]$

8. Show that $\dfrac{\tan A}{\sec A-1}+\dfrac{\tan A}{\sec A+1}=2\cos ecA$. Soln:

$$\frac{\tan A}{\sec A-1}+\frac{\tan A}{\sec A+1}=\frac{\tan A(\sec A+1)+\tan A(\sec A-1)}{(\sec A-1)(\sec A+1)}$$

$$=\frac{2\tan A\sec A}{\sec^2 A-1}=\frac{2\tan A\sec A}{\tan^2 A}=\frac{2\sec A}{\tan A}=2\cos ecA$$

9. Show that $\cot A-\tan A=\dfrac{2\cos^2 A-1}{\sin A\cos A}$. Soln:

$\cot A-\tan A$

$$=\frac{\cos A}{\sin A}-\frac{\sin A}{\cos A}$$

$$=\frac{\cos^2 A-\sin^2 A}{\sin A\cos A}=\frac{\cos^2 A-(1-\cos^2 A)}{\sin A\cos A}$$

$$=\frac{2\cos^2 A-1}{\sin A\cos A}$$

10. Show that $\dfrac{\cos A}{1-\tan A}-\dfrac{\sin^2 A}{\cos A-\sin A}=\cos A+\sin A$. Soln:

$$\frac{\cos A}{1-\tan A}-\frac{\sin^2 A}{\cos A-\sin A}$$

$$=\frac{\cos A}{\dfrac{\cos A-\sin A}{\cos A}}-\frac{\sin^2 A}{\cos A-\sin A}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}-\frac{\sin^2 A}{\cos A-\sin A}$$

$$=\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}=\frac{(\cos A+\sin A)(\cos A-\sin A)}{\cos A-\sin A}$$

$$=\cos A+\sin A=R.H.S$$

11. Show that $1-\dfrac{\cos^2 A}{1+\sin A}=\sin A$. Soln:

$$1-\frac{\cos^2 A}{1+\sin A}=1-\frac{(1-\sin^2 A)}{1+\sin A}$$

$$=1-\frac{(1-\sin A)(1+\sin A)}{1+\sin A}$$

$$=1-(1-\sin A)=\sin A=R.H.S$$

12. Show that $\sqrt{\dfrac{1-\cos A}{1+\cos A}}=\dfrac{\sin A}{1+\cos A}$. Soln :

$$\sqrt{\frac{1-\cos A}{1+\cos A}}=\sqrt{\frac{(1-\cos A)}{(1+\cos A)}\frac{(1+\cos A)}{(1+\cos A)}}$$

$$=\sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}=\sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$=\frac{\sin A}{1+\cos A}$$

13. Show that $\dfrac{\sin A\tan A}{1-\cos A}=1+\sec A$. Soln:

$$\frac{\sin A\tan A}{1-\cos A}=\frac{\sin A\tan A(1+\cos A)}{(1-\cos A)(1+\cos A)}$$

$$=\frac{\sin A\tan A(1+\cos A)}{\sin^2 A}$$

$$=\frac{\tan A(1+\cos A)}{\sin A}=\frac{(1+\cos A)}{\cos A}=\frac{1}{\cos A}+1$$

$$=\sec A+1$$

14. Show that $\dfrac{\cos A\cot A}{1-\sin A}=1+\operatorname{cosec} A$. Soln: Same as problem 13.

15. Show that $\sec^4 A-\tan^4 A=1+2\tan^2 A$. Soln:

$$\sec^4 A-\tan^4 A=(\sec^2 A-\tan^2 A)(\sec^2 A+\tan^2 A)$$

$$=(\sec^2 A+\tan^2 A)=1+\tan^2 A+\tan^2 A$$

$$=1+2\tan^2 A$$

16. Show that $(\sin A+\cos ecA)^2+(\cos A+\sec A)^2=\tan^2 A+\cot^2 A+7$ Soln:

$$(\sin A+\cos ecA)^2+(\cos A+\sec A)^2$$

$$=\sin^2 A+\cos ec^2 A+2\sin A\cos ecA+\cos^2 A+\sec^2 A+2\sec A\cos A$$

$$=\sin^2 A+\cos ec^2 A+\cos^2 A+\sec^2 A+4$$

$$=\cos ec^2 A+\sec^2 A+5$$

$$=1+\cot^2 A+1+\tan^2 A+5$$

$$=\cot^2 A+\tan^2 A+7=R.H.S$$

17. Show that $\sec^6 A-\tan^6 A=1+3\tan^2 A+3\tan^4 A$. Soln:

$$\sec^6 A-\tan^6 A=(\sec^2 A)^3-(\tan^2 A)^3$$

$$=(\sec^2 A-\tan^2 A)(\sec^4 A+\sec^2 A\tan^2 A+\tan^4 A)$$

$$=(\sec^4 A+(1+\tan^2 A)\tan^2 A+\tan^4 A)$$

$$=\left[(1+\tan^2 A)^2+(1+\tan^2 A)\tan^2 A+\tan^4 A\right]$$

$$=\left[1+\tan^4 A+2\tan^2 A+\tan^2 A+\tan^4 A+\tan^4 A\right]$$

$$=\left[1+3\tan^2 A+3\tan^4 A\right]$$

18. Show that $\dfrac{\sin A}{\cot A+\cos ecA}=2+\dfrac{\sin A}{\cot A-\cos ecA}$. Soln:

$$L.H.S=\frac{\sin A}{\cot A+\cos ecA}$$

$$=\frac{\sin A(\cot A-\cos ecA)}{(\cot A+\cos ecA)(\cot A-\cos ecA)}$$

$$=\frac{\cos A-1}{-1}=1-\cos A$$

$$R.H.S=2+\frac{\sin A}{\cot A-\cos ecA}$$

$$=2+\frac{\sin A(\cot A+\cos ecA)}{(\cot A-\cos ecA)(\cot A-\cos ecA)}$$

$$=2+\frac{\cos A+1}{-1}=2-\cos A-1=1-\cos A$$

19. Show that $(\cos ecA-\sin A)(\sec A-\cos A)(\tan A+\cot A)=1$. Soln:

$$(\cos ecA-\sin A)(\sec A-\cos A)(\tan A+\cot A)$$

$$=\left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{1+\cot^2 A}{\cot A}\right)$$

$$=\left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\cos ec^2 A}{\cot A}\right)$$

$$=\left(\frac{\cos A}{1}\right)\left(\frac{\sin A}{1}\right)\left(\frac{\cos ec^2 A}{\cot A}\right)$$

$$=\cos A\sin A\frac{\sin A}{\cos A\sin^2 A}=1$$

20. Show that $\dfrac{1}{\sec A+\tan A}-\dfrac{1}{\cos A}=\dfrac{1}{\cos A}-\dfrac{1}{\sec A-\tan A}$. Soln:

$$\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$$

iff

$$\frac{1}{\sec A+\tan A}+\frac{1}{\sec A-\tan A}=\frac{1}{\cos A}+\frac{1}{\cos A}$$

$$L.H.S=\frac{\sec A-\tan A+\sec A+\tan A}{\sec^2 A-\tan^2 A}$$

$$=2\sec A=\frac{2}{\cos A}=\frac{1}{\cos A}+\frac{1}{\cos A}$$

21. Show that $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \sec^2 A - \sec^2 B$, Soln:

$\tan^2 A - \tan^2 B = \sec^2 A - 1 - (\sec^2 B - 1)$

$= \sec^2 A - \sec^2 B = \dfrac{1}{\cos^2 A} - \dfrac{1}{\cos^2 B}$

$= \dfrac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$

$= \dfrac{(1 - \sin^2 B) - (1 - \sin^2 A)}{\cos^2 A \cos^2 B}$

$= \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

22. Show that $\dfrac{\sec A - 1}{\sec A + 1} = (\cot A - \cos ecA)^2$. Soln:

$\dfrac{\sec A - 1}{\sec A + 1} = \dfrac{1 - \cos A}{1 + \cos A}$

$= \dfrac{\cos ecA - \cot A}{\cos ecA + \cot A} [\mathit{Dividing\ num.\quad denom.\quad by}\quad \sin A]$

$= \dfrac{(\cos ecA - \cot A)^2}{(\cos ecA + \cot A)(\cos ecA - \cot A)}$

$= \dfrac{(\cos ecA - \cot A)^2}{(\cos ec^2 A - \cot^2 A)}$

$= (\cos ecA - \cot A)^2$

23. Show that $\dfrac{\tan A + \sin A}{\tan A - \sin A} = \dfrac{\sec A + 1}{\sec A - 1}$. Soln:

$\dfrac{\tan A + \sin A}{\tan A - \sin A}$

$= \dfrac{\sec A + 1}{\sec A - 1} [\mathit{Dividing\ num.\quad denom.\quad by}\quad \sin A]$

24. If $\tan^2 A = 1 + 2\tan^2 B$ show that $\cos^2 B = 2\cos^2 A$. Soln:

$\tan^2 A = 1 + 2\tan^2 B$

$\Rightarrow \sec^2 A - 1 = 1 + 2(\sec^2 B - 1)$

$\Rightarrow \sec^2 A = 2\sec^2 B$

$\Rightarrow \cos^2 B = 2\cos^2 A$

25. If $\begin{matrix} x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \\ x\sin\theta - y\cos\theta = 0 \end{matrix}$ show that $x^2 + y^2 = 1$

$x\sin\theta = y\cos\theta........(A)$

$\therefore y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow y\cos\theta(1-\cos^2\theta) + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow y\cos\theta - y\cos^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow y\cos\theta = \sin\theta\cos\theta$

$\Rightarrow y = \sin\theta$

$from\,(A) \quad x = \cos\theta$

$\Rightarrow x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

26. If $a\sin^2\theta + b\cos^2\theta = c \;\&\quad p\sin^2\theta + q\cos^2\theta = r$, prove that $(b-c)(r-p) = (c-a)(q-r)$ Soln:

$given \quad a\sin^2\theta + b\cos^2\theta = c \;\& \quad p\sin^2\theta + q\cos^2\theta = r$

$\therefore a\sin^2\theta + b(1-\sin^2\theta) = c$

$\Rightarrow (a-b)\sin^2\theta = c-b$

$\Rightarrow \sin^2\theta = \dfrac{c-b}{a-b}$

$Similarly \quad \sin^2\theta = \dfrac{r-q}{p-q}$

$\therefore \dfrac{c-b}{a-b} = \dfrac{r-q}{p-q}$

$\Rightarrow cp - cq - bp + bq = ra - rb - qa + qb$

$\Rightarrow cp - cq - bp = ra - rb - qa$

$\Rightarrow br - bp - cr + cp = -cr + ra - qa + cq$

$\Rightarrow (b-c)(r-p) = (c-a)(q-r)$

27. Eliminate A from the equations

$x\sin A - y\cos A = \sqrt{x^2+y^2} \qquad and \qquad \dfrac{\cos^2 A}{a^2} + \dfrac{\sin^2 A}{b^2} = \dfrac{1}{x^2+y^2}$ Soln:

$Squaring \quad the\ first\ equation$

$(x\cos A + y\sin A) = 0$

$\Rightarrow \dfrac{\cos A}{-y} = \dfrac{\sin A}{x}$

$\Rightarrow \cos A = \dfrac{-y}{\sqrt{x^2+y^2}}, \sin A = \dfrac{x}{\sqrt{x^2+y^2}}$

$Substituiting\ in \sec ond\ equation$

$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

28. Eliminate A from the equations $\tan A - \cot A = a$ *and* $\sin A + \cos A = b$

$$a^2 = (\tan A - \cot A)^2 = \tan^2 A + \cot^2 A - 2\tan A \cot A$$

$$= 1 + \sec^2 A + 1 + \cos ec^2 A - 2$$

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} + 4$$

Soln: $\therefore a^2 - 4 = \dfrac{\cos^2 A + \sin^2 A}{\sin^2 A \cos^2 A} = \dfrac{1}{\sin^2 A \cos^2 A}$

$$b^2 = 1 + 2\sin A \cos A$$

$$\therefore b^2 - 1 = 2\sin A \cos A$$

$$\Rightarrow (b^2 - 1)^2 = 4\sin^2 A \cos^2 A$$

$$\therefore (b^2 - 1)^2 (a^2 - 4) = 4$$

29. If $\cos^2 A - \sin^2 A = \tan^2 B$, show that $\cos^2 B - \sin^2 B = \tan^2 A$ Soln:

$$\cos^2 A - \sin^2 A = \tan^2 B$$

$$\Rightarrow 2\cos^2 A - 1 = \tan^2 B$$

$$\Rightarrow 2\cos^2 A = \tan^2 B + 1$$

$$\Rightarrow 2\cos^2 A = \sec^2 B$$

$$\Rightarrow 2\cos^2 B = \sec^2 A$$

$$\Rightarrow 2\cos^2 B = 1 + \tan^2 A$$

$$\Rightarrow 2\cos^2 B - 1 = \tan^2 A$$

$$\Rightarrow \cos^2 B - \sin^2 B = \tan^2 A$$

30. If $\sec A + \tan A = x,$ show that $\sin A = \dfrac{x^2 - 1}{x^2 + 1}$ Soln:

$$\frac{1}{x} = \frac{1}{\sec A + \tan A} = \frac{\sec A - \tan A}{(\sec A + \tan A)(\sec A - \tan A)} = \sec A - \tan A$$

$$\therefore x + \frac{1}{x} = 2\sec A$$

and $\quad x - \dfrac{1}{x} = 2\tan A$

Dividing $\dfrac{2\tan A}{2\sec A} = \dfrac{x - \frac{1}{x}}{x + \frac{1}{x}}$

$$\Rightarrow \sin A = \frac{x^2 - 1}{x^2 + 1}$$

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