

Prove that $(\csc \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) = 1$

1.

$$\begin{aligned} & (\csc \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) \\ &= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{1 + \tan^2 \theta}{\tan \theta} \right) \\ &= \frac{\cos^2 \theta}{\sin \theta} \frac{\sin^2 \theta}{\cos \theta} \frac{\sec^2 \theta}{\tan \theta} = \frac{\sin^2 \theta \cdot \cos \theta}{\sin^2 \theta \cdot \cos \theta} = 1 \end{aligned}$$

Prove that $(1 + \cot \theta + \tan \theta)(\sin \theta - \cos \theta) = \left(\frac{\sec \theta}{\csc^2 \theta} - \frac{\csc \theta}{\sec^2 \theta} \right)$

2.

$$\begin{aligned} & (1 + \cot \theta + \tan \theta)(\sin \theta - \cos \theta) \\ &= \left(\frac{\tan \theta + 1 + \tan^2 \theta}{\tan \theta} \right) (\sin \theta - \cos \theta) \\ &= \left(\frac{\tan \theta + 1 + \tan^2 \theta}{\sin \theta \sec \theta} \right) \left(\frac{\sec \theta - \csc \theta}{\csc \theta \sec \theta} \right) \\ &= \cos^2 \theta (\tan \theta + 1 + \tan^2 \theta) (\sec \theta - \csc \theta) \\ &= \cos^2 \theta \cdot \sin^2 \theta (\tan \theta \csc^2 \theta + \csc^2 \theta + \tan^2 \csc^2 \theta) (\sec \theta - \csc \theta) \\ &= \cos^2 \theta \cdot \sin^2 \theta (\sec \theta \csc \theta + \csc^2 \theta + \sec^2 \theta) (\sec \theta - \csc \theta) \\ &= \cos^2 \theta \cdot \sin^2 \theta (\sec^3 \theta - \csc^3 \theta) \\ &= \left(\frac{\sec \theta}{\csc^2 \theta} - \frac{\csc \theta}{\sec^2 \theta} \right) \end{aligned}$$

Prove that $(\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 = \cot^2 \theta + \tan^2 \theta + 7$

3.

$$\begin{aligned} & (\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 \\ &= \sin^2 \theta + \csc^2 \theta + 2 + \cos^2 \theta + \sec^2 \theta + 2 \\ &= \sin^2 \theta + \cos^2 \theta + 1 + \cot^2 \theta + 1 + \tan^2 \theta + 4 \\ &= \cot^2 \theta + \tan^2 \theta + 7 \end{aligned}$$

Prove that $2 \sec^2 \theta - \sec^4 \theta - 2 \csc^2 \theta + \csc^4 \theta = \cot^4 \theta - \tan^4 \theta$

4.

$$\begin{aligned} & 2 \sec^2 \theta - \sec^4 \theta - 2 \csc^2 \theta + \csc^4 \theta = 2(\sec^2 \theta - \csc^2 \theta) + \csc^4 \theta - \sec^4 \theta \\ &= 2(\sec^2 \theta - \csc^2 \theta) + (\csc^2 \theta - \sec^2 \theta)(\csc^2 \theta + \sec^2 \theta) \\ &= (\csc^2 \theta - \sec^2 \theta)(\csc^2 \theta + \sec^2 \theta - 2) \\ &= (\cot^2 \theta - \tan^2 \theta)(2 + \cot^2 \theta + \tan^2 \theta - 2) \\ &= (\cot^2 \theta - \tan^2 \theta)(\cot^2 \theta + \tan^2 \theta) \\ &= \cot^4 \theta - \tan^4 \theta \end{aligned}$$

Prove that $(\tan A + \csc B)^2 - (\cot B - \sec A)^2 = 2 \tan A \cot B (\sec B + \csc A)$

5.

$$\begin{aligned}
 & (\tan A + \operatorname{cosec} B)^2 - (\cot B - \sec A)^2 \\
 &= \tan^2 A + \operatorname{cosec}^2 B + 2 \tan A \operatorname{cosec} B - \cot^2 B - \sec^2 A + 2 \cot B \sec A \\
 &= \tan^2 A - \sec^2 A + \operatorname{cosec}^2 B - \cot^2 B + 2 \tan A \operatorname{cosec} B + 2 \cot B \sec A \\
 &= -1 + 1 + 2 \tan A \operatorname{cosec} B + 2 \cot B \sec A \\
 &= 2 \tan A \cot B (\tan B \operatorname{cosec} B + \cot A \sec A) \\
 &= 2 \tan A \cot B (\sec B + \operatorname{cosec} A)
 \end{aligned}$$

6. Prove that $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$

$$\begin{aligned}
 & \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\sin \theta + 1 - \cos \theta}{\sin \theta - 1 + \cos \theta} \\
 &= \frac{(\sin \theta - \cos \theta) + 1}{(\sin \theta + \cos \theta) - 1} = \frac{((\sin \theta - \cos \theta) + 1)((\sin \theta + \cos \theta) + 1)}{((\sin \theta + \cos \theta) - 1)((\sin \theta + \cos \theta) + 1)} \\
 &= \frac{\sin^2 \theta - \cos^2 \theta + 2 \sin \theta + 1}{1 + 2 \sin \theta \cos \theta - 1} \\
 &= \frac{2 \sin^2 \theta + 2 \sin \theta}{2 \sin \theta \cos \theta} = \frac{1 + \sin \theta}{\cos \theta}
 \end{aligned}$$

7. Prove that $\frac{\cos \theta \operatorname{cosec} \theta - \sin \theta \sec \theta}{\cos \theta + \sin \theta} = \operatorname{cosec} \theta - \sec \theta$

$$\begin{aligned}
 & \frac{\cos \theta \operatorname{cosec} \theta - \sin \theta \sec \theta}{\cos \theta + \sin \theta} \\
 &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta \sin \theta (\cos \theta + \sin \theta)} \\
 &= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{\cos \theta \sin \theta (\cos \theta + \sin \theta)} \\
 &= \frac{\cos \theta - \sin \theta}{\cos \theta \sin \theta} = \operatorname{cosec} \theta - \sec \theta
 \end{aligned}$$

8. Prove that $\left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \cos^2 \theta \cdot \sin^2 \theta = \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \cos^2 \theta \sin^2 \theta}$

$$\begin{aligned}
 & \left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \cos^2 \theta \cdot \sin^2 \theta \\
 &= \left(\frac{\sec^2 \theta}{\sec^4 \theta - 1} + \frac{\operatorname{cosec}^2 \theta}{\operatorname{cosec}^4 \theta - 1} \right) \cos^2 \theta \cdot \sin^2 \theta \\
 &= \frac{\sin^2 \theta}{\tan^2 \theta (\sec^2 \theta + 1)} + \frac{\cos^2 \theta}{\cot^2 \theta (\operatorname{cosec}^2 \theta + 1)} \\
 &= \frac{\cos^2 \theta \cdot \cos^2 \theta}{1 + \cos^2 \theta} + \frac{\sin^2 \theta \cdot \sin^2 \theta}{1 + \sin^2 \theta} \\
 &= \frac{\cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^2 \theta (\cos^2 \theta + \sin^2 \theta)}{2 + \cos^2 \theta \sin^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(\cos^2 \theta + \sin^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta}{2 + \cos^2 \theta \sin^2 \theta} \\
 &= \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \cos^2 \theta \sin^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 9. \sec^4 \theta - \sec^2 \theta &= \sec^2 \theta (\sec^2 \theta - 1) = (1 + \tan^2 \theta) \tan^2 \theta \\
 &= \tan^4 \theta + \tan^2 \theta
 \end{aligned}$$

10. If $3 \sec^4 \theta + 8 = 10 \sec^2 \theta$, find the value(s) of $\tan \theta$

$$\begin{aligned}
 \text{Let } t &= \sec^2 \theta \\
 \therefore 3 \sec^4 \theta + 8 &= 10 \sec^2 \theta \\
 \Rightarrow 3t^2 - 10t + 8 &= 0 \\
 \Rightarrow t &= \frac{4}{3}, 2 \\
 \Rightarrow \sec \theta &= \frac{2}{\sqrt{3}}, \sqrt{2}
 \end{aligned}$$

Now find $\tan \theta$ yourself!

11. If $\sin \theta = \frac{m^2+2mn}{m^2+2mn+2n^2}$ prove that $\tan \theta = \frac{m^2+2mn}{2mn+2n^2}$. $\cos \theta =$

$$\begin{aligned}
 \sqrt{1 - \frac{(m^2+2mn)^2}{(m^2+2mn+2n^2)^2}} &= \frac{\sqrt{(m^2+2mn+2n^2)^2 - (m^2+2mn)^2}}{m^2+2mn+2n^2} \\
 &= \frac{\sqrt{(m^2+2mn+2n^2+m^2+2mn)(m^2+2mn+2n^2-m^2-2mn)}}{m^2+2mn+2n^2} \\
 &= \frac{\sqrt{2(m^2+2mn+n^2).2n^2}}{m^2+2mn+2n^2} \\
 &= \frac{2n(m+n)}{m^2+2mn+2n^2} \\
 \therefore \tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\frac{m^2+2mn}{m^2+2mn+2n^2}}{\frac{2n(m+n)}{m^2+2mn+2n^2}} = \frac{m^2+2mn}{2mn+2n^2}
 \end{aligned}$$

12. If $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$, find $\cos \theta + \sin \theta$

$$\begin{aligned}
 \cos \theta + \sin \theta \\
 = \cos \theta + \frac{\cos \theta}{\sqrt{2}+1} &= \cos \theta + (\sqrt{2}-1) \cos \theta = \sqrt{2} \cos \theta
 \end{aligned}$$

Solved Trigonometric problems for students of 10th grade

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