

Set Theory, Relations and Functions

Problem Set

1. Use properties of set to prove that for all sets A and B $A - (A \cap B) = A - B$
2. Prove that for any sets A, B and C, $(A - B) \cap (C - B) = (A \cap C) - B$
3. Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the values of m and n.
{Answer: m=6, n=3}
4. Show that $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = B \cap C'$
5. If $X = \{8^n - 7n - 1 | n \in N\}$ and $Y = \{49n - 49 | n \in N\}$. Then
 - a. X is a subset of Y
 - b. Y is a subset of X
 - c. X equals Y
 - d. $X \cap Y = \emptyset$ {Hint: using induction show that $8^n - 7n - 1$ is always divisible by 49}
6. If $A = \emptyset$, find the number of elements in the set $P(P(P(A)))$, where $P(A)$ denotes the power set of A. Also write one element of $P(P(P(A)))$
7. If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, prove that B=C.
8. If P is the set of all prime numbers and $T = \{4^{4^n} + 2 | n \in N\}$, find $P \cap T$.
9. Prove that $[A \cap (B \cup C)] \cap [A' \cup (B' \cap C')] = \emptyset$.
10. If $B \subset A$ show that $A' \subset B'$.
11. If $T = \{xy = 1 | x, y \text{ are real numbers}\}$ and $P = \{x = y | x, y \text{ are real numbers}\}$. Find $|P \cap T|$.
12. If $Z = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \dots \dots \dots \}$, that is Z is the set of all integers and $P_0 = \{3n | n \in Z\}, P_1 = \{3n + 1 | n \in Z\}, P_2 = \{3n + 2 | n \in Z\}$ show that $Z = P_0 \cup P_1 \cup P_2$
13. If $A = \{4, 8, 12, 16, 20, 24, 28, 32, \dots \dots \dots \}$ and $B = \{6, 12, 18, 24, 30, 36, \dots \}$. Find $A \cap B$
14. If $P_1 \subset P_2 \subset P_3 \subset \dots \subset P_n$. Find $\bigcup_{i=1}^n P_i$ and $\bigcap_{i=1}^n P_i$
15. Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
16. Find the domain and range of the relations described below.

Author: VINOD KUMAR SINGH

E-mail: maths.programming@gmail.com

Website: <http://kolkatamaths.yolasite.com>

- i. $R = \{(x, y) \mid y = x + \frac{6}{x} \text{ where } x, y \in N \text{ and } x \leq 6\}$
- ii. $R = \{(x, y) \mid y = x + 1 \text{ where } x, y \in N\}$
- iii. $R = \{(x, y) \mid y = x^2 \text{ where } x, y \in N \text{ and } x \leq 100\}$

17. Is the relation R from \mathbb{R} to \mathbb{R} defined as $R = \{(x, |x| - 1) \mid x \text{ is real}\}$ is a function? Justify your answer.
18. If for any sets A and B $|A| = m$ and $|B| = n$. How many relations can be defined from A to B? How many functions can be defined from A to B? Deduce that $2^{mn} > n^m$.
19. If $R = \{(x, y) \mid x, y \text{ are both integers and } x^2 + y^2 = 64\}$. Find the domain and range of the relation R
20. Is the relation R from $\mathbb{R} \setminus \{0\}$ to \mathbb{R} defined as

$$R = \left\{ \left(x, \frac{|x|}{x} \right) \mid x \text{ is real and non-zero} \right\}$$
 is a function? Justify your answer.

Find the domain and range of the relation. Plot a rough sketch.

21. For the real functions f described below, find the domain and range
 - i. $f(x) = x$
 - ii. $f(x) = \frac{1}{x}$
 - iii. $f(x) = e^x$
 - iv. $f(x) = [x]$
 - v. $f(x) = x^2 + 1$
 - vi. $f(x) = x^2 + 2x + 1$
 - vii. $f(x) = x - |x - 7|$
 - viii. $f(x) = ||x| - 1| + |x|$
22. For the real functions f described below, find the domain
 - i. $f(x) = \frac{x^2 + 2x + 1}{x^2 - x - 6}$
 - ii. $f(x) = \frac{x^2 + 2x + 1}{x^2 + x + 1}$
 - iii. $f(x) = \sqrt{x - 1}$
 - iv. $f(x) = \frac{1}{\sqrt{x-1}}$
 - v. $f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}}$
 - vi. $f(x) = 7x^{101} + x - 3$

23. If $f(x) = x$, find $f^n(x)$.
24. If $f(x) = x + 1$, find $f^4(x)$ and $f^n(x)$.
25. If $A = \{a, b, c\}$ and $B = \{2, 5\}$. How many one-one (injective) functions can be defined from A to B?

Author: VINOD KUMAR SINGH

E-mail: maths.programming@gmail.com

Website: <http://kolkatamaths.yolasite.com>

Feel free to contact me for any queries on maths.programming@gmail.com

Facebook: <http://facebook.com/prime.maths>

Blogger: <http://mathsvinu.blogspot.in>

YouTube: <https://www.youtube.com/c/PrimeMaths>