

Integration

1. Integrate $\int \frac{dx}{x^2\sqrt{x+1}}$ Ans: $-\frac{\sqrt{x+1}}{x} + \frac{1}{2} \log \left\{ \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1} \right\} + c$ Hint: put

$\sqrt{x+1} = t^2$ this will reduce the integration to $\int \frac{dt}{(t^2-1)^2}$, now apply partial fraction.

2. Integrate $\int \frac{dx}{(x^2+x)\sqrt{x+1}}$ Hint: put $\sqrt{x+1} = t^2$ Same as above.

3. Integrate $\int \frac{dx}{(x^3+x^2)\sqrt{x+1}}$ Hint: $\int \frac{dx}{(x^3+x^2)\sqrt{x+1}} =$

$\int \frac{(x+1-x)dx}{(x^3+x^2)\sqrt{x+1}} = \int \frac{dx}{x^2\sqrt{x+1}} - \int \frac{dx}{(x^2+x)\sqrt{x+1}}$ Now use the above two integrals.

4. Integrate $\int \frac{(x+1)dx}{(x+2)(x+3)^{3/2}}$ Hint: $\int \frac{(x+1)dx}{(x+2)(x+3)^{3/2}} =$

$$\int \frac{(x+2-1)dx}{(x+2)(x+3)^{3/2}} = \int \frac{dx}{(x+3)^{3/2}} - \int \frac{dx}{(x+2)(x+3)^{3/2}} = \int \frac{dx}{(x+3)^{3/2}} - \int \frac{dx}{(x+2)(x+3)\sqrt{(x+3)}}$$

Now put $x+3 = t^2$ in both integrals and integrate.

5. $\int \sqrt{\frac{1+x^2}{x^2-x^4}}dx$ Ans: $\tan^{-1} \sqrt{\frac{1+x^2}{1-x^2}} + \frac{1}{2} \log \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} + c$ Hint:

$\int \sqrt{\frac{1+x^2}{x^2-x^4}}dx = \int \frac{1}{x} \sqrt{\frac{1+x^2}{1-x^2}}dx$ Now put $\frac{1+x^2}{1-x^2} = y^2$ and the integrand will reduce to $\int \frac{2y^2}{(y^2-1)(y^2+1)}dy$

6. Integrate $\int \frac{x^2+1}{x\sqrt{4x^2+1}}dx$

7. Integrate $\int \frac{1+x^{1/4}}{1+x^{1/2}}dx$ Hint: Put $x = t^4$ Ans:

$$\frac{4}{3}x^{3/4} + 2\sqrt{x} - 4x^{1/4} - 2\log(\sqrt{x}+1) + 4\tan^{-1} x^{1/4} + c$$

8. Integrate $\int x^{-2/3}(1+x^{1/2})^{-5/3}dx$: Hint Put $x = \frac{1}{t}$. In general for the integration

$\int x^m(a+bx^n)^pdx$, if p is not an integer and $p + \frac{m+1}{n}$ becomes an integer the

substitution is $x = \frac{1}{t}$

9. Integrate $\int \frac{(1+x \cos \alpha)}{(1+2x \cos \alpha + x^2)^{3/2}} dx$ Put $1+2x \cos \alpha + x^2 = t^2$ then

$(x + \cos \alpha)dx = dt$, clearly $x = \sqrt{t^2 - \sin^2 \alpha} - \cos \alpha$ therefore

$$dx = \frac{dt}{\sqrt{t^2 - \sin^2 \alpha}} \text{ and}$$

$$(1+x \cos \alpha) = 1 + \cos \alpha \left(-\cos \alpha + \sqrt{t^2 - \sin^2 \alpha} \right) = \sin^2 \alpha + \cos \alpha \sqrt{t^2 - \sin^2 \alpha}$$

$$\text{Substituting we get } \int \frac{(1+x \cos \alpha)}{(1+2x \cos \alpha + x^2)^{3/2}} dx = \int \frac{\sin^2 \alpha + \cos \alpha \sqrt{t^2 - \sin^2 \alpha}}{t^2 \sqrt{t^2 - \sin^2 \alpha}} dt$$

$$\text{let } k = \sin \alpha \text{ and } l = \cos \alpha \text{ therefore } \int \frac{\sin^2 \alpha + \cos \alpha \sqrt{t^2 - \sin^2 \alpha}}{t^2 \sqrt{t^2 - \sin^2 \alpha}} dt =$$

$$\int \frac{k^2 + l \sqrt{t^2 - k^2}}{t^2 \sqrt{t^2 - k^2}} dt = k^2 \int \frac{dt}{t^2 \sqrt{t^2 - k^2}} + l \int \frac{dt}{t^2} = \sqrt{1 - \frac{k^2}{t^2}} - \frac{l}{t} + c = \frac{\sqrt{t^2 - k^2} - l}{t} + c =$$

$$\frac{\sqrt{1+2x \cos \alpha + x^2 - \sin^2 \alpha} - \cos \alpha}{\sqrt{1+2x \cos \alpha + x^2}} + c = \frac{\sqrt{(x+\cos \alpha)^2 + \sin^2 \alpha - \sin^2 \alpha} - \cos \alpha}{\sqrt{1+2x \cos \alpha + x^2}} + c = \frac{x}{\sqrt{1+2x \cos \alpha + x^2}} + c$$

10. Integrate $\int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx$ Hint:

$$\int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx = \int \frac{x^4 - 1}{x^2 \cdot x \sqrt{x^2 + \frac{1}{x^2} + 1}} dx = \int \frac{\left(x - \frac{1}{x^3} \right)}{\sqrt{x^2 + \frac{1}{x^2} + 1}} dx \text{ Now put}$$

$$x^2 + \frac{1}{x^2} = t \text{ and integrate. Ans: } \frac{\sqrt{x^4 + x^2 + 1}}{x} + c$$

Author-Vinod Singh

Education : M.Sc Pure Mathematics'09, (Calcutta University) First Class.

B.Sc Mathematics Honours'07, (St. Xavier's kolkata) First Class.

Special interest in Algebra, Algebraic and Analytical Number Theory,

Cryptography, Algebraic Topology and Geometry.

<http://kolkatamaths.yolasite.com>

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Best of luck!



Call us on +91-9038126497

Mail-maths.kolkata@gmail.com/math.vinu@gmail.com

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