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Probability

1. Probability of an impossible event is _______.

Ans: Zero, since impossible events can never occur so number of favourable outcomes is zero.

2. Probability of an event E + probability of an event not E = _____________.

Ans: 1, since the event E and not E are mutually exclusive and exhaustive events one of them will certainly occur.

3. Probability of a sure event is _________.

Ans: 1, since it will always occur whenever the random experiment is performed i.e, the total number of favourable outcomes is equal to the total number of possible outcomes.

4. If a die is thrown once, find the probability of getting a perfect outcomes.

Ans: Sample Space $S=\{1,2,3,4,5,6\}$
Favourable outcomes = $\{1,4,\}$
Therefore the required probability = $\frac{2}{6}=\frac{1}{3}$

5. A card is drawn from a well shuffled deck of playing cards. Find the probability that the card drawn is either a king or a queen.

Ans: Since one card is drawn at a time, the drawn card can be any one of the 52 playing cards.
Hence the total possible outcomes is 52.
Let A be the event that the drawn card is either a king or a queen. There are 4 kings and 4 queens in a deck of playing cards.
So, favourable outcomes = 4+4 = 8.
Therefore the required probability = $P(A)=\frac{8}{52}=\frac{2}{13}$

6. If one coin of Re. 1 and another coin of Rs. 2 are tossed simultaneously, find the probability of getting at least one head.

Ans. Bascially there are 2 coins which are being tossed. If S is the sample space
S={(H,H),(H,T),(T,H),(T,T)}.
number of favourable outcomes for the given event is (H,H),(H,T),(T,H)
Therefore the required probability = $\frac{3}{4}$

7. If an integer is chosen between 0 and 100, then find the probability that its is not divisible by 7.

Ans: Let A be the required event i.e., the integer chosen is not divisible by 7.
The selected integer can be any integer between 0 to 100 (both inclusive), so total possible outcomes is 101.
Now, $P(A)=1-P(A')$
A' is the event where the selected integer is divisible by 7.
The integers which are divisible by 7 in the given range is 0,7,14,21,......., 98.

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Let the total number of such integers be n.

$t_n = a + (n-1)d$ Using A.P and observe that a=0 ,d=7 and nth term is 98

therefore $98 = 0 + (n-1)7 \Rightarrow n = 15$

$$P(A) = 1 - P(A')$$
$$\Rightarrow P(A) = 1 - \frac{15}{101}$$
$$\Rightarrow P(A) = \frac{101-15}{101}$$
$$\Rightarrow P(A) = \frac{86}{101}$$

8. A bag contains 4 red balls and 6 black balls. If a ball is taken out at random, find the probability of getting a black ball.

Ans: Probability of getting a black ball is = $\frac{total\ number\ of\ blk\ balls}{total\ number\ of\ balls} = \frac{6}{10} = \frac{3}{5}$

9. A number x is chosen at random from the numbers -4, -3, -2, -1, 0, 1, 2, 3, 4. Find the probability that $|x| < 2$.

Ans: Total number of possible outcomes is 9 since x can be any one of the numbers -4, -3, -2, -1, 0, 1, 2, 3, 4.

If $|x| < 2$ then x has to be -1, 0, 1

Therefore the required probability = $\frac{3}{9}$

10. A number is selected at random from the numbers 9, 9, 9, 9, 7, 7, 7, 5, 5, 3. Find the probability that the selected number is equal to the average of given numbers.

Ans: Total number of possible outcomes is 10. Mean of the given numbers is

$$\frac{9+9+9+9+7+7+7+5+5+3}{10} = \frac{70}{10} = 7$$

So, if the selected number is 7, it will be equal to the average of given numbers.

Therefore number of favourable outcomes = 3 {7,7,7}

Required probability = $\frac{3}{10}$

11. If a coin is tossed 3 times, find the probability of getting

a) at least two heads

b) no heads

c) exactly two heads

d) atmost two heads

Ans: Let S be the sample space. Since the coin is being tossed 3 times. Total number of possible outcomes will be $2^3 = 8$ .We are listing S below

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$$\begin{array}{l} H\ H\ H \\ H\ H\ T \\ H\ T\ H \\ H\ T\ T \\ T\ H\ H \\ T\ H\ T \\ T\ T\ H \\ T\ T\ T \end{array}$$

P(at least two heads) = $\frac{4}{8}=\frac{1}{2}$. Since favourable outcomes for this event are HHH,HHT,HTH,THH,

P(no heads) = $\frac{1}{8}$. only one favourable outcome TTT

P(exactly two heads) = $\frac{3}{8}$. Since favourable outcomes for this event are HHT,HTH,THH

P(atmost two heads) = $\frac{7}{8}$. Since favourable outcomes for this event are all the elemenst of the sample space except HHH.

12. A bag contains 16 balls out of which x are green. If one ball is drawn at random, what is the probability that it will be a green ball? If 8 more green balls are put int the bag, the probability of drawing a green ball will be twice of the above. Find x.

Ans: Total number of possible outcomes is 16 since one ball can be drawn from a lot on 16 balls in 16 ways.
Let A be the probability that the drawn ball is green in the first case.
Number of favourable outcomes for the event A is x

therefore $P(A)=\frac{x}{16}$

Let B be the event that the drawn ball is green after adding 8 balls.
Total number of possible outcomes in this case is 16+8 = 24
Number of favourable outcomes for the event A is x+8

therefore $P(B)=\frac{x+8}{24}$

According to the problem P(B) = 2P(A)

$$\Rightarrow \frac{x+8}{24}=\frac{2x}{16}$$
$$\Rightarrow 16x+128=48x$$
$$\Rightarrow x=\frac{128}{32}=4$$

13. A box contains 20 balls bearing numbers 1,2,3,4,.........20. A ball is drawn at random from the box. What is the probability that the number on the ball is (i) an odd number (ii) divisible by 2 or 3 (iii) a prime number (iv) not divisible by 10?

Ans: i) Total number of possible outcomes is 20 since one ball can be drawn from a lot on 20 balls in 20 ways.
Number of odd balls is 10.

therefore P(a ball of odd number is drawn) = $\frac{10}{20}=\frac{1}{2}$

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ii) numbers which is divisible by 2 or 3 are 2,3,4,6,8,9,10,12,14,15,16,18,20

therefore P(divisible by 2 or 3) = $\frac{13}{20}$

iii) prime numbers in the list are 2,3,5,7,11,13,17,19

therefore P(a prime number) = $\frac{8}{20}=\frac{2}{5}$

iv) numbers which are divisible by 10 are 10, 20

P(not divisible by 10) = 1- P(divisible by 10)

$$\Rightarrow P(\textit{not divisible by } 10)=1-\frac{2}{20}$$

$$\Rightarrow P(\textit{not divisible by } 10)=1-\frac{1}{10}$$

$$\Rightarrow P(\textit{not divisible by } 10)=\frac{9}{10}$$

14. Find the probability that a leap year, selected at random, will have 53 Sundays.

Ans: A leap year has 366 days. Each year regardless whether it is leap or not has 52 weeks (52 x 7 = 364 days) So there will be 52 Sundays in each year. If there has to be 53 Sundays it has to fall on the one of the 2 days left (366-364). There are 7 week days. Probability that a given day will be sunday is $\frac{1}{7}$. Since there is 2 extra days left the required probability is $\frac{2}{7}$.

15. Find the probability that the month February may have 5 Wednesdays in (a) a leap year. (b) a non-leap year.

Ans: (a) In a leap year the month of February has 29 days. Number of full weeks in February is 4 (4 x 7 = 28). So there will be 4 Wednesday one in each week. If there has to be 5 Wednesday it has to fall on the 1 day left (29-28). There are 7 week days. Probability that a given day will be Wednesday is $\frac{1}{7}$

In a non-leap year there cannot be 5 Wednesdays, so the probability is 0. Since there is no extra day left for the 5th Wednesday.

16. N coins are tossed simultaneously. Find the probability that there will be a) exactly N heads b) atmost N-1 heads.

Ans: Each of the coin has 2 possibilites. Therefore number of elemensts in the sample space is

$$\overbrace{2\times2\times2.........\times2}^{N\ number\ of\ 2's}=2^N$$

Now exactly N heads can occur in only one way when all the coins shows up head. So there is exactly one favourable outcome HHHH......H (N-times).

P(exactly N heads) = $\frac{1}{2^N}$

P(atmost N-1 heads) = 1 – P(exactly N heads)

$$= 1-\frac{1}{2^N}$$

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17. Two dice are thrown simultaneously. Find the probability that the sum of the numbers on the faces of two dice is odd.

Ans: Total number of possible outcomes when two dice are rolled = $6^2=36$
Let A be the required event. Favourable outcomes for the event A is (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5).
So number of favourable outcomes = 18.
Therefore the required probability = $\frac{18}{36}=\frac{1}{2}$
Alternatively the number of favourable outcomes for the event A can be calculated as follows. Remember that sum of two numbers is odd if and only if one of them is odd. Odd + Even = Odd Since 2k+1 + 2n = 2(k+n)+1, $k, n \in \mathbb{Z}$
Consider one of the dice and fixed it, it has 3 odd numbers which when coupled with the 3 odd numbers of the other dice will sum to a odd number. In this case total number of ways of getting odd number is 3x3 = 9 and now swapping the dice gives another set of 9 odd pairs. Therefore total favourable outcomes for the event A is 9+9 = 18 ! Easy compared to the enumerative method.

18. A die is thrown twice. What is the probability that (i) 5 will come up at least once (ii) 5 will not come up either time?

Ans: Total number of possible outcomes when a die is thrown twice = $6^2=36$
In this problem we will not be using the enumerative method to find out the total number of favourable outcomes!
Since 5 has to come up at least once. Consider the following outcome (5, X). X can be any one of the numbers 1,2,3,4,5,6. So there is 6 possibilites. Now swapping the position will give another 6 possibilites but (5,5) is repeated in both the cases so total favourable outcomes = 6+6-1 = 11.

Therefore the required probability = P(5 will come up at least once) = $\frac{11}{36}$
If you like, list down all the outcomes! Or rather visit our fb page http://facebook.com/kolkatamaths

P(5 will not come up either time) = 1 - P(5 will come up at least once)

$= 1-\frac{11}{36}$

$= \frac{25}{36}$

19. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is divisible by 6 or 8?

Ans: Total possible outcomes is 100.
Let n be the total number of numbers between 1 and 100 that is divisible by 6.
$t_n=a+(n-1)d$ Using A.P and observe that a=6 ,d=6 and nth term is 96
therefore $96=6+(n-1)6 \Rightarrow n=16$
Similarly number of numbers between 1 and 100 that is divisible by 8 is 12.

Now observe that there are numbers which are divisible by both 6 and 8 hence counted twice if we just add 16 and 12. So we have to subtract the numbers counted twice. For example 24, 48 are both divisible by 6 and 8.
To find such numbers observe that an integer which is both divisible by 6 and 8 is divisible by LCM of 6 and 8. which is 24.
Now number of numbers which is divisible by 24 between 1 and 100 are 24,48,72,96 (you can use the A.P formula)
Therefore number of integers divisible by 8 0r 6 is 16+12-4 = 24

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required probability = $\frac{24}{100}=\frac{6}{25}$

20. Two dice are rolled simultaneously. Find the probability that the sum of the numbers on the two dice is less than 11.

Ans: Total number of elements in the sample space is 36. Let A be the required event. Then A' is the event that the sum of the numbers on the two dice is greater than equal to 11. favourable outcomes for A' are (5,6), (6,5) and (6,6).
P(A) = 1- P(A')

$$\Rightarrow P(A)=1-\frac{3}{36}$$

$$\Rightarrow P(A)=\frac{33}{36}=\frac{11}{12}$$

Observe that if you calculate P(A) directly, you have to list down all the outcomes for which the sum is less than 11. i.e., you have to list 33 outcomes! That will be tiresome!