

ISC 2009
MATHEMATICS
(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper, They must NOT start writing during this time.)

SECTION A-Answer Question 1(compulsory) and five other questions.

Section B and Section C- Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and squared paper are provided. Slide rule may be used.

SECTION-A
Question 1

i) If $M(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ show that $M(x)M(y) = M(x+y)$

ii) The lines $x-2y+6=0$ and $2x-y-10=0$ intersect at the point A. Find the equation of the line making an angle 45° with the positive direction of the x-axis and passing through the point A.

iii) Find the equations of the tangents to the parabola $y^2+12x=0$ from the point (3,8).

iv) Find the derivative of $\sin x^2$ with respect to x^3 .

v) Evaluate the following integral $\int \frac{e^{2x}}{2+e^x} dx$

vi) Evaluate the following limit: $\lim_{x \rightarrow \pi/4} \frac{1-\tan x}{\cos 2x}$

vii) Two horses are considered for a race. The probability of selection of the first horse is $\frac{1}{4}$ and that of the second is $\frac{1}{3}$. What is the probability that:

(a) both of them will be selected.

- (b) only one of them will be selected.
- (c) none of them will be selected

viii) The mean weight of 70 students in a class is 60 kg. The mean weight of the girls in the class is 53 kg and that of the boys is 70.5 kg. Find the number of girls in the class.

ix) If $(-2+\sqrt{-3})(-3+2\sqrt{-3})=a+ib$, find the real numbers a and b . With these values of a and b , also find the modulus of $a+ib$.

x) Solve the following differential equation: $(x \cos y)dy=e^x(x \log x+1)dx$

Question 2

(a) By using properties of determinants, prove that:

$$\begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4 \sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4 \sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4 \sin 2x \end{vmatrix} = 2 + 4 \sin 2x$$

(b) Solve the following linear equations using matrix method:

$$\begin{aligned} x+y+z &= 9 \\ 2x+5y+7z &= 52 \\ 2x+y-z &= 0 \end{aligned}$$

Question 3

(a) Prove that the following equation represent a pair of straight lines. Find their point of intersection and the angle between them:
 $2x^2+7xy+3y^2+2(4x+7y+4)=0$

(b) P, Q and R represent switches in 'on' positions and P^1, Q^1 and R^1 represent switches in 'off' positions. Construct a switching circuit representing the polynomial $PR+Q(Q^1+R)(P+QR)$.

Question 4

(a) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that
 $x^2 - y^2 - z^2 + 2xy\sqrt{1-x^2}$

(b) Using a suitable substitution find the derivative of $\tan^{-1} \frac{4\sqrt{x}}{1-4x}$

Question 5

(a) It is given that the Rolle's theorem holds good for the function $f(x)=x^3+ax^2+bx$, $x \in [1,2]$ at the point $x=\frac{4}{3}$ Find the values of a and b .

(b) A wire of length 20m is available to fence off a flower bed in the form of a sector of a circle. What must be the radius of the circle, if we wish to have a flower bed with the greatest possible area?

Question 6

Integrate the following

(a) (i) Evaluate: $\int_0^{\frac{\pi}{2}} \log(\tan x) dx$

(ii) Evaluate: $\int_0^5 (x + \frac{1}{2}) dx$

- (b) Draw a rough sketch of the curve $x^2+y=9$ and find the area enclosed by the curve, the x-axis and the lines $x+1=0$ and $x-2=0$.

Question 7

- (a) An examination of 8 applicants for a clerical post was taken by a firm. The marks obtained by the applicants in the Reasoning and Aptitude tests are given below

Applicant	:A	B	C	D	E	F	G	H
Reasoning Test:	20	28	15	60	40	80	20	15
Aptitude Test:	30	50	40	20	10	60	30	35

Calculate the Spearman's coefficient of rank correlation from the data given above.

- (b) If the two regression lines of a bivariate distribution are $4x-5y+33=0$ and $20x-9y-107=0$.
- (i) calculate x^- and y^- , the arithmetic means of x and y respectively.
- (ii) estimate the value of x when $y = 7$
- (iii) find the variance of y when $\sigma_x = 3$

Question 8

- (a) Bag A contains 5 white and 4 black balls, and bag B contains 7 white and 6 black balls. One ball is drawn from the bag A and without noticing its colour, is put in the bag B. If a ball is then drawn from bag B, find the probability that it is black in colour.

- (b) An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 104 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the article manufactured will not be defective.

Question 9

- (a) If $z = \frac{13-5i}{4-9i}$, prove by using De Moivre's theorem that $z^6 = -8i$
- (b) Solve the following differential equation for a particular solution:
 $dy = (5x-4y)dx$; when $y=0$ and $x=0$.

SECTION B

Question 10

- (a) Find the equation of the plane which contains the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and

is perpendicular to the plane $x+2y+z=12$.

(b) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-6z-4=0$, $x+2y+2z=0$ and whose centre lies on the plane $2x-y+z=1$.

Question 11

(a) Find the area of a parallelogram whose diagonals are determined by the vectors $\vec{a}=3\vec{i}+\vec{j}-2\vec{k}$ and $\vec{b}=\vec{i}-3\vec{j}+\vec{k}$

(b) (i) Prove by vector method that the diameter of a circle will subtend a right angle at a point on its circumference.

(ii) If \vec{a} , \vec{b} and \vec{c} represent the position of the points with co-ordinates $(2,-10,2)$, $(3,1,2)$ and $(2,1,3)$ respectively, find the value of $\vec{a} \times (\vec{b} \times \vec{c})$.

Question 12

(a) The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least 6 successes.

(b) An insurance company insured 4000 doctors, 8000 teachers and 12000 engineers. The probabilities of a doctor, a teacher and an engineer dying before the age of 58 years are 0.01, 0.03 and 0.05 respectively. If one of the insured person dies before the age of 58 years, find the probability that he is a doctor.

NOTE: Section C is not for science students!!! If you found any mistake let me know. This is a board paper, I have just posted for the benefit for the students.