

ISC 2008  
MATHEMATICS  
(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper, They must NOT start writing during this time.)

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SECTION A-Answer Question 1(compulsory) and five other questions.

Section B and Section C- Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets[ ].

Mathematical tables and squared paper are provided. Slide rule may be used.

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SECTION-A  
Question 1

- i) Find  $x$  and  $y$ , if  $x+y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$  and  $x-y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$
- ii) Find the equation of the straight line through origin and passing through the intersection of lines  $x-2y+3=0$  and  $3x+5y+6=0$ .
- iii) Find the equations of the normal to the ellipse  $5x^2+3y^2=137$  at the point where the ordinate is 2.
- iv) If  $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$ , find  $\frac{dy}{dx}$ .
- v) Evaluate  $\int \frac{x^2}{x^2-4} dx$
- vi) Find the equation of tangents to the hyperbola  $3x^2-y^2=3$  which are perpendicular to the line  $x+3y=2$ .
- vii) In a single throw of two dice, find the probability of getting a total of at most 9.
- viii) If the standard deviation of the numbers 2,3,11 and  $x$  is  $3\frac{1}{2}$ , find the value of  $x$ .

ix) Find the value of x and y, given that  $(x+iy)(2-3i)=4+i$ .

x) Solve the following differential equation:

$$(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2.$$

Question 2

(a) Prove that:

$$\begin{pmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{pmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$$

(b) If  $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$ , find  $A^{-1}$  and hence solve the following system of

linear equations:

$$x+2y-3z=-4$$

$$2x+3y+2z=2$$

$$3x-3y-4z=11$$

Question 3

(a) (i) Show that the second degree equation  $x^2-5xy+4y^2+x+2y-2=0$  represent a pair of straight lines.

(ii) Find the equation of the individual lines and their point of intersection.

(b) (i) Write down the Boolean expression corresponding to the switching circuit given below:-

$$!(A+A).(B+D).(C+A).(C+D)!$$

(ii) Simplify the expression and construct the switching circuit for the simplified expression.

Question 4

(a) Solve for x:  $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ .

(b) Find  $\frac{dy}{dx}$  if  $y = \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$

Question 5

(a) Use Lagrange's mean value theorem to determine a point P on the curve  $y = \sqrt{x-2}$  defined in the interval  $[2,3]$  where the tangent is parallel to the chord joining the end points on the curve.

(b) An open box with a square base is to be made out of a given quantity of

cardboard whose area is  $c^2$  square units. Show that the maximum volume of the box is  $\frac{c^3}{6\sqrt{3}}$  cubic units.

Question 6

(a) (i) Evaluate:  $\int_0^9 f(x)dx$ , where  $f(x)$  is defined by

$$\sin x : \text{if } 0 \leq x \leq \frac{\pi}{2}$$

$$1 : \frac{\pi}{2} \leq x \leq 5$$

$$e^{x-5} : 5 \leq x \leq 9$$

(b) Draw a rough sketch of the curve  $y^2+1=x$ ,  $x \leq 2$ . Find the area enclosed by the curve and the line  $x=2$ .

Question 7

(a) The data for marks in Physics and History obtained by ten students are given below:-

Marks in Physics : 15 12 8 8 7 7 7 6 5 3

Marks in History : 10 25 17 11 13 17 20 13 9 15

Using this data:

Calculate the Karl Pearson's coefficient of correlation between the marks in Physics and History obtained by the 10 students.

(b) (i) Find the line of regression in which the Physics is taken as the independent variable.

(ii) A candidate had scored 10 marks in physics but was absent from the History test. Estimate his probable score for the latter test.

Question 8

(a) There are 3 urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consists of 2 red balls and 1 black ball?

(b) The probability that a teacher will give an unannounced test during any class meeting is  $\frac{1}{5}$ . If a student is absent twice, find the probability that the student will miss at least one test.

Question 9

(a) If the ratio  $\frac{z-i}{z-1}$  is purely imaginary, prove that the point  $z$  lies on the circle whose centre is the point  $\frac{1}{2}(1+i)$  and radius is  $\frac{1}{\sqrt{2}}$ .

b) Solve:  $(x^2+y^2)dx-2xydy=0$ , given that  $y=0$ , when  $x=1$ .

## SECTION B

### Question 10

(a) Find the coordinates of the point where the line joining the points (1,-2,3) and (2,-1,5) cuts the plane  $x-2y+3z=19$ . Hence, find the distance of this point from the point (5,4,1).

(b) If  $A=(-1,4,-3)$  is one end of a diameter AB of the sphere  $x^2+y^2+z^2-2y+2z-15=0$  then find the coordinates of the other end point B.

### Question 11

(a) A small industrial concern used three raw materials A,B and C in its manufacturing process. The prices of the materials was as shown below:-

Commodity.....Price in Rs.in the year 1995.....Price in Rs.in the year 2005

A .....	4 .....	5
B.....	60.....	57
C.....	36.....	42

Using 1995 as the base year, calculate a simple aggregate price index for 2005.

(b) Coded monthly sales figures of a particular brand of T.V. for 18 months commencing January 1,200 as follows:-

Year.....	jan	feb	mar	apr	may	jun	jul	aug	sept	oct	nov	dec
2005.....	18...	16...	23...	27...	28...	19...	31...	29...	35...	27...	28...	24
2006.....	24...	28...	29...	30...	29...	22						

Calculate six monthly moving averages and display these and the original figures on the same graph using the same axes for both.

### Question 12

(a) Find  $\vec{a} \cdot \vec{b}$  if  $|\vec{a}| = 2$ ,  $|\vec{b}| = 5$  and  $|\vec{a} \times \vec{b}| = 8$ .

(b) Given  $\vec{a} = i - 2j + k$ ,  $\vec{b} = 2i + j + k$ , and  $\vec{c} = i + 2j - k$ ,

Find :  $\vec{a} \times (\vec{b} \times \vec{c})$

NOTE: Section C is not for science students!!! If you found any mistake let me know. This is a board paper, I have just posted for the benefit of the students. In this paper in the question 3(b)i the boolean diagram is not present i am learning how to draw figures in pdf format! instead i have written the boolean expression itself from which you can draw the figure.