ISC 2007
MATHEMATICS
(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper, They must NOT start writing during this time.)

SECTION A-Answer Question 1(compulsory) and five other questions.

Section B and Section C- Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in bracktes[ ].

Mathematical tables and squared paper are provided. Slide rule may be used.

SECTION-A

Question 1

i) If A= $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, show that A^2-3I=2A

ii) Find the equation of the straight line passing through the intersection of straight lines 2x+3y=1 and 3x+4y=6 and perpendicular to the line 5x-2y=7.

iii)Find the equations of the parabola having focus at (3,-4) and directrix as x+y=2.

iv) If e^{x+y}= xy, show that $\frac{dy}{dx}=\frac{y(1-x)}{x(y-1)}$.

v)Solve $\int \frac{1}{xcos^2(1+logx)}dx$

vi) Find the equation of the ellipse whose minor axis is 4 and which has a distance of 6 units between its foci.

vii) Kamal and Monica appera for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Monika's selection is $\frac{1}{5}$.Find the probability that only one of them is selected.

viii)Five students scored marks as follows:

8,10,15,20,22

Find the S.D of the above distribution.

ix) Find the modulus and amplitude of the complex number $\frac{2+3i}{3+2i}$

x) Solve the following differential equation:

tany dx+ $sec^2 y$ tanx dy=0.

Question 2

(a) Using the properties of determinants, show that:

$$\begin{pmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ \end{pmatrix} = (x+y+z)^2$$

(b) Solve the following system of linear equations using matrices:

x+y+z=6

2x+y-z=1

x-y+z=2

Question 3

(a) (i) Find the value of 'k' so that the second degree equation $12x^2$-10xy+ky^2+14x-5y+2=0 may represent a pair of straight lines.For this value of 'k' find the angle between the lines represented by the above equation.

(b) Show that the bisectors of the angles between the pair of straight lines $11x^2$-16xy-y^2=0 are parallel and perpendicular to x+2y+5=0.

Question 4

(a)Prove that $2tan^{-1}\frac{1}{3} + cot^{-1}4 = tan^{-1}\frac{16}{13}$.

(b) If $\sin y$=xsin$(a+y)$, show that

$\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$

Question 5

(a) Verify Lagrange's Mean value theorem for the function f(x)=$3x^2$-5x+1 defined in the interval [2,5].

(b) Show that the semi vertical angle of a cone of maximum volume and of given slant height is $tan^{-1}\sqrt{2}$.

Question 6

(a) Evaluate: $\overset{\frac{\pi}{4}}{\underset{0}{}} \int log(1+tanx)dx$

(b) Find the area enclosed by the curves y^2=x and y^2=4-3x.

Question 7

(a) Calculate the coefficient of correlation between X and Y from the following data using Karl Pearson's method.

X: 1 2 3 4 5

Y: 2 5 3 8 7

(b) Given two regression lines 4x+3y+7=0 and 3x+4y+8=0, determine

(i) The regression line of y on x

(ii) The regression line of x on y

(ii) The coefficient of correlation.

Question 8

(a) Thw bag A contains 3 white and 2 black balls while bag B contains 2 white and 5 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is white?

(b) Assuming that on an average one telephone out of ten is busy, seven telephone numbers are randomly selected and called. Find the probability that three of them will be busy.

Question 9

(a) Prove that $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8 = -2^8$ by using De Moivre's theorem.

b) Solve the differential equation

$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$

SECTION B

Question 10

(a) Find the equation of the plane through the point (1,2,3) and perpendicular to the planes x+y+2z=3 and 3x+2y+z=4.

(b) Prove that the plane x+2y-z=4 cuts the sphere x^2+y^2+z^2-x+z-2=0 in a circle whose radius is unity.

Question 11

(a)If $\vec{a}$,$\vec{b}$,$\vec{c}$ are three vectors, show that $(\vec{a}+\vec{b}).(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})$=2[$\vec{a}\vec{b}\vec{c}$]

(b)Find a unit vector perpendicular to the vectors 4i+3j+k and 2i-j+2k. Determine the sine of the angle between these vectors.

Question 12

(a)!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

(b) !!!

It's really boring to write the statistics question!!!!!!!

NOTE: Section C is not for science students!!! If you found any mistake let me know. This is a board paper, I have just posted for the benifit of the students. Please avoid attempting statistics question.