

ISC 2006
MATHEMATICS
(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper, They must NOT start writing during this time.)

SECTION A-Answer Question 1(compulsory) and five other questions.

Section B and Section C- Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets[].

Mathematical tables and squared paper are provided. Slide rule may be used.

SECTION-A

Question 1

i) Solve for x and y:

$$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2 \begin{pmatrix} 2x \\ 3y \end{pmatrix} = 3 \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

ii) A straight line is parallel to the x-axis and passes through the intersection of the lines $x + 2y + 1 = 0$ and $y = x + 7$. Find the equation of the straight line.

iii) A straight line $2x+y+p=0$ is a focal chord of the parabola $y^2=-8x$. Find the value of p.

iv) If $y = e^{\sin x^2}$ find $\frac{dy}{dx}$.

v) Evaluate $\int x^2(e^{x^3})\cos(2e^{x^3})dx$

vi) Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$ and whose foci are the points $(\pm 2, 0)$.

vii) The probability of A,B and C solving a problem are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try and solve the problem simultaneously, find the probability that only one of them will solve it.

viii) One set of 100 observations has the mean as 15 and another set of 150 observations has the mean as 16. Find the mean of 250 observation by combining the two sets of given observation.

ix) Express $\frac{1-2i}{2+i} + \frac{3+i}{2-i}$ in the form $a+ib$.

x) Solve the following differential equation:
 $x(x^2 - x^2y^2)dy + y(y^2 + x^2y^2)dx = 0$.

Question 2

(a) Show that:

$$\begin{pmatrix} 1 & 1 & 1 \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha^3 & \beta^3 & \gamma^3 \end{pmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha\beta + \beta\gamma + \alpha\gamma)$$

(b) Find the adjoint of the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$ and hence find A^{-1} .

Question 3

(a) Show that the equation $x^2 - 3xy + 2y^2 + 3x - 5y + 2 = 0$ represent two straight lines. Find the angle between them.

(b) Find the equation of the bisectors of the angle between the pair of lines given by $(4ax + 3by)^2 = 3(3bx - 4ay)^2$.

Question 4

(a) Show that: $\sin^{-1}\left(\frac{1}{\sqrt{17}}\right) + \cos^{-1}\left(\frac{9}{\sqrt{85}}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

(b) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $(1-x^2)\frac{dy}{dx} - xy = 1$.

Question 5

(a) Verify Rolle's theorem for the function $f(x) = e^{2x}(\sin 2x - \cos 2x)$ defined in the interval $[\frac{\pi}{8}, \frac{5\pi}{8}]$

(b) Find the volume of the largest cone that can be inscribed in a sphere of radius R.

Question 6

(a) Prove that $\int_0^{2\pi} \frac{x \cos x}{1+\cos x} dx = 2\pi^2$.

(b) The region bounded by the curve $y^2 = x(x-1)^2$, the x-axis and the lines $x=1, x=0$, is rotated through four right angles about the x-axis. Calculate the

volume of the solid of revolution so formed.

Question 7

- (a) Calculate the Spearman's rank correlation coefficient between the advertisement cost and sales from the following data:

Advertisement cost: 39 65 62 90 82 75 25 98 36 78

(Rs. in thousands)

Sales(Rs. in lakhs): 47 53 58 86 62 68 60 91 51 84

- (b) The following table shows the sales and advertisement expenditure or a firm:

Sales(Rs. in crores) Advertisement(Rs. in crores)

Mean 40 6

Standard deviation 10 1.5

Coefficient of correlation = $\gamma=0.9$

Estimate the likely sales for a proposed advertisement expenditure of Rs. 10 crores.

Question 8

- (a) A and B throw two dice each. If A gets a sum of 9 on his two dice, then find the probability of B getting a higher sum.

- (b) A card is drawn at random from a pack of 52 playing cards. What is the probability that the card drawn is neither a spade nor a queen?

Question 9

- (a) Find the cube roots of -27 and show that the sum of the cube roots is equal to zero.

- b) Solve the differential equation

$$ydx - (x + 2y^2)dy = 0$$

SECTION B

Question 10

- (a) Find the equation of the two planes passing through the points (0,4,-3) and (6,-4,3), if the sum of their intercepts on the three axes is zero.

- (b) Show that the equation to a sphere passing through three points (2,0,0), (0,2,0) and (0,0,2) and having its centre on the plane $2x + 3y + 4z = 27$ is $x^2 + y^2 + z^2 - 6x - 6y - 6z + 8 = 0$

Question 11

- (a) The vectors $-2\mathbf{i}+4\mathbf{j}+4\mathbf{k}$ and $-4\mathbf{i}-2\mathbf{k}$ represent the diagonals BD and AC of a

parallelogram ABCD. Find the area of the parallelogram.

(b) The vectors $i+3j, 5k$ and $\lambda i-j$ are coplanar. Find the value of λ .

Question 12

Useless statistics question no need to attepmt this question!!!!!!!

NOTE: Section C is not for science students!!! If you found any mistake let me know. This is a GUESS paper, I have just posted for the benifit of the students to help them in preparing for their board exam.

Vinod Kumar Singh

9874059952

<http://sites.google.com/site/mathsvinu>