INDIAN SCHOOL CERTIFICATE

Question Paper, 2002

MATHEMATICS

Time : 3 hrs. *Marks : 100*

Answer Question 1 from Part I and ten questions from Part II, choosing four questions from Section A, two questions from Section B, two questions from Section C and two questions from Section D.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and squared paper are provided. Slide rule may be used.

PART – I

Answer all questions.

Question 1.

(*i*) In the binomial expansion of $(\sqrt[3]{3} + \sqrt{2})^5$, find the term which does not contain irrational expressions. [3]

(*ii*) A sphere with centre (R, – R, R) and radius R passes through the point (1, – 1, 4). Find the centre and radius of the sphere. [3]

(*iii*) Find the eccentricity of the ellipse.

$$\frac{(x-3)^2}{8} + \frac{(y-4)^2}{6} = 1.$$ [3]

(*iv*) Find the equation of the plane passing through (1, 2, 3) and perpendicular to the straight line $\frac{x}{-2} = \frac{y}{-4} = \frac{z}{3}$. [3]

(*v*) There are 10 persons who are to be seated around a circular table. Find the probability that two particular persons will always sit together. [3]

(*vi*) If $y = e^x \log \tan 2x$, find $\frac{dy}{dx}$. [3]

(*vii*) Using vectors, show that the medians of a triangle meet at a point. [3]

(*viii*) Evaluate $\int_0^{\pi/4} \frac{2 \cos 2x}{1 + \sin 2x}\, dx$. [3]

(*ix*) Simplify

$(1 - 3\omega + \omega^2)(1 + \omega - 3\omega^2)$. [3]

(*x*) Find the inverse of the matrix. $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$. [3]

PART – II

SECTION A

Answer any four questions.

Question 2.

(a) Given that [4]

$(1+x)^n = C_0 + C_1x + C_2x^2 \ldots + C_nx^n$, and $C_0 + C_1 + C_2 + \ldots + C_n = 1024$. Find the value of n and hence find the term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ which contains x^{11}.

(b) Prove by the method of mathematical induction that $3^{2n-1} + 3^n + 4$ is divisible by 2 for all $n \in N$. [3]

Question 3.

(a) Using properties of determinant, find the value of the following determinant : [3]

$$D = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$$

(b) Using Cramer's rule, solve the following system of equations : [4]

$$x - y = 1,$$
$$x + y = -6,$$
$$x + y - 2z = 3.$$

Question 4.

(a) Find the equation of the straight line passing through the intersection of the lines $2x + 3y = 1$ and $3x + 2y = 2$ and making equal intercepts on the axes. [4]

(b) The equation of the directrix of the parabola is $3x + 2y + 1 = 0$. The focus is (2, 1). Find the equation of the parabola. [3]

Question 5.

(a) The plane $x + 2y + 2z + 7 = 0$ cuts the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ in a circle. Find the centre and the radius of the circle. [4]

(b) Show that $\sin^{-1}\frac{\sqrt{3}}{2} + 2\tan^{-1}\frac{1}{\sqrt{3}} = \frac{2\pi}{3}$. [3]

Question 6.

An open box with a square base is to be made out of a given quantity of cardboard whose area is c^2 units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{6}}$ units. [7]

Question 7.

(*a*) Evaluate $\int_0^1 x \tan^{-1} x \, dx$. [3]

(*b*) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the axis of x. [4]

SECTION – B

Answer any two questions.

Question 8.

(*a*) The vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. Given that $|\vec{a}| = |\vec{b}|$. Find the values of x and y. [4]

(*b*) Given that :

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$. Find the vector $\vec{a} \times (\vec{b} \times \vec{c})$. [3]

Question 9.

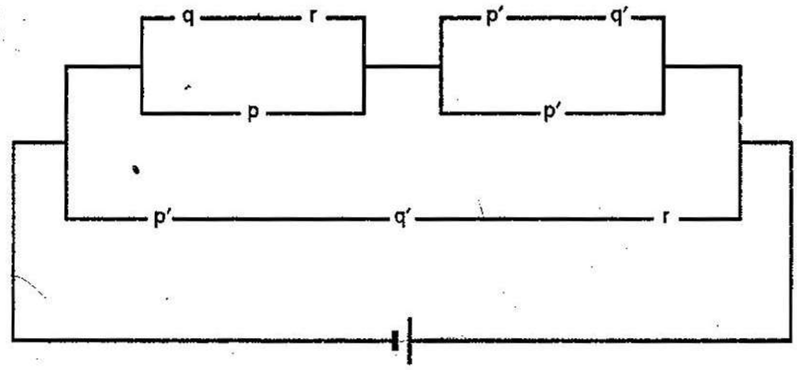
(*a*) Let an ordered pair of real numbers be called a complex number, and let addition $\oplus$ of complex number be defined by $(a, b) \oplus (c, d) = (a + c, b + d)$. [4]

Show that the set of complex numbers together with binary operation $\oplus$ forms an abelian group.

(*b*) Show that the set I of all integers does not form a group under the operation defined as $a * b = a - b$ for every $a, b \in$ I. [3]

Question 10.

Write Boolean function corresponding to the following switching circuit network : [7]



Use the laws of Boolean algebra to simplify the circuit. Construct the network for the simplified circuit.

SECTION – C

Attempt any two questions.

Question 11.

The marks obtained by nine students in Physics and Mathematics are given below : [7]

Physics :	35	23	47	17	10	43	9	6	28
Mathematics :	30	33	45	23	8	49	12	4	31

Calculate Spearman's coefficient of rank correlation and interpret the result.

Question 12.

(*a*) In a certain test, the 30 scores were grouped as follows : [7]

Scores :	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Frequency :	2	2	7	10	6	2	1

Calculate the standard deviation.

(*b*) Taking 1975 as the base year, with an index number 100, calculate an index number for 1979, based on weighted average of price relatives from the table given below : [3]

Commodity :	A	B	C	D
Weight :	30	15	25	30
Price per unit in 1975 :	20	10	5	40
Price per unit in 1979 :	24	20	30	40

Question 13.

An insurance company insured 6000 scooter drives, 3000 car drivers and 9000 truck drivers. The probability of an accident involving a scooter, a car and a truck is 0.02, 0.06 and 0.30 respectively. One of the insured persons meets with an accident. Find the probability that he is a car driver. [7]

SECTION – D

Attempt any two questions.

Question 14.

(*a*) Sketch in the complex plane the set of points z satisfying. [4]

$$\frac{|z-3|}{|z+1|} = 3.$$

(*b*) Solve :

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$ [3]

Question 15.

(*a*) Given that $\dfrac{2\sqrt{3}\cos 30^\circ - 2\,i\sin 30^\circ}{\sqrt{2}\,(\cos 45^\circ + i\sin 45^\circ)} = A + iB.$

Find the values of A and B. [4]

(*b*) Solve :

$$e^y(1+x^2)\,dy - \frac{x}{y}\,dx = 0.$$ [3]

Question 16.

(*a*) Using Demoivre's theorem, find the least value of n for which the expression $\left(\dfrac{\sqrt{3}+3i}{2\sqrt{3}}\right)^{2n-1}$ is purely real. [4]

(*b*) Solve :

$$\cos^2 x\,\frac{dy}{dx} + y = \tan x.$$ [3]