ISC 2010 GUESS PAPER

MATHEMATICS

(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper, They must NOT start writing during this time.)

SECTION A-Answer Question 1(compulsory) and five other questions.

Section B and Section C- Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in bracktes[ ].

Mathematical tables and squared paper are provided. Slide rule may be used.

SECTION-A

Question 1

i) If A= $\begin{pmatrix} 5 & 3 \\ 12 & 7 \end{pmatrix}$, show that A^2-12A-I=O.

ii) Find the equation of the straight line passing through the intersection of straight lines 2x+3y=3 and 3x+4y=1 and parallel to the line 5x+y=3.

iii)Find the equations of the parabola having focus at (-2,1) and directrix as 6x-3y=8.

iv) If $y = tan^{-1}\frac{3x-x^3}{1-3x^2}$, show that $\frac{dy}{dx} = \frac{3}{2}$ at x=1.

v)Solve $\int \frac{x^2}{\sqrt{1+x}}dx$

vi) Find the equation of the ellipse whose foci are (±2,3) and whose semi-minor axis is $\sqrt{5}$.

vii) Kamal and Monica appera for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Monika's selection is $\frac{1}{5}$.Find the probability that only one of them is selected.

viii)Five students scored marks as follows:

1.7,1.9,2.3,2.7,2.2

Find the S.D of the above distribution.

ix) Find the modulus and amplitude of the complex number $\frac{1}{1+cos\theta+isin\theta}$

x) Solve the following differential equation:

$\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$

Question 2

(a) Using the properties of determinants, show that:

$$\begin{pmatrix} a^2 & a & 1 \\ cos(nx) & cos(n+1)x & cos(n+2)x \\ sin(nx) & sin(n+1)x & sin(n+2)x \end{pmatrix} = (a^2-2acosx+1)sinx$$

(b) Solve the following system of linear equations using matrices:

x+y+2z=4

2x-y+3z=9

3x-y-z=2

Question 3

(a) (i) Find the value of 'k' so that the second degree equation $2x^2$+xy-$3y^2$+4x+ky-6=0 may represent a pair of straight lines.

(b) A,B,C represent switches in an 'on' position and $A^{'}$,$B^{'}$,$C^{'}$, represent switches in an 'off' position;construct a switching circuit representing the polynomial [B(B+A)][C($B^{'}$+C)]. Use Boolean algebra to show that the above circit is equivalent to a switching circuit,in which when B and C are 'on', the light is 'on'.

Question 4

(a)Find the value of $cos^{-1}x+cos^{-1}(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2})$, $\frac{1}{2}\leq x\leq 1$.

(b) If $x=\frac{1+t}{t^3}$ and y=$\frac{3}{2t^2}+\frac{2}{t}$, show that

$xy^{'3}$=$1+y^{'}$.

Question 5

(a) Use Lagrange's mean value theorem to determine a point P on the curve y=$\sqrt{x^2-4}$ defined in the interval [2,4] where the tangent is parallel to the chord joining the end points on the curve.

(b) Show that the minimum and maximum value of y=$x(logx)^2$ is 0 and $4e^{-2}$ respectively.

Question 6

(a) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x+cos^4x}dx$

(b) Compute the area of the figure bounded by the straight lines x=0 and x=2 and the curves y=2^x and y=2x-x^2.

Question 7

(a) Calculate the coefficient of correlation between X and Y from the following data using Karl Pearson's method.

X: 13 14 15 16 17

Y: 14 20 15 20 19

(b) Given two regression lines x+6y-6=0 and 3x+2y-10=0, determine

(i) The regression line of y on x

(ii) The regression line of x on y

(ii) The coefficient of correlation.

Question 8

(a) Thw bag A contains 5 white and 3 black balls while bag B contains 3 white and 4 black balls. One of the bags is chosen at random and a ball is drawn from it. What is the probability that the ball is black?

(b) Assuming that on an average one telephone out of ten is busy, seven telephone numbers are randomly selected and called. Find the probability that three of them will be busy.

Question 9

(a) (i) If x=$cos\theta + isin\theta$, y=$cos\phi + isin\phi$ and m and n are integers show that, $\frac{x^m}{y^n} + \frac{y^n}{x^m}$=$2cos(m\theta - n\phi)$

(ii) if ω is cube root of unity show that $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$=32

b) Solve the differential equation

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(x^2+1)^2}$

SECTION B

Question 10

(a) Find the equation of the plane through the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z+1}{5}$ and perpendicular to the plane x-y+z+2=0.

(b) Find the equation of the sphere whose centre is the point (2,3,1) and which touches the plane x+y+z=0.

Question 11

(a)Show that $|\vec{a} \times i|^2 + |\vec{a} \times j|^2 + |\vec{a} \times k|^2 = 2a^2$.

(b) Using vector method show that if the mid-points of the consecutive sides of any quadrilateral are connected by straight lines, prove that the resulting quadrilateral is a parallelogram.

Question 12

(a)Three six-faced fair dice are thrown together. Find the probability that the sum of the numbers appearing on the dice is k.($1 \leq k \leq 8$)

(b) A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is six. Show that the probability that it is actually a six is $\frac{3}{8}$ (Hint use Bayes' theorem)

NOTE: Section C is not for science students!!! If you found any mistake let me know. This is a GUESS paper, I have just posted for the benifit of the students to help them in preparing for their board exam.

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