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MATHEMATICS

(Maxumum Marks : 100)

(Time allowed: Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time.)

Section A- Answer Question 1 (compulsory) and five other questions.

Section B and Section C – Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A (80 Marks)

Question 1. [10x3]

(i) Find the matrix X for which:

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

Solution:

$$X = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$$

(ii) Solve for x, if: $\tan(\cos^{-1} x) = \frac{2}{\sqrt{5}}$

Solution:

Given, $\tan(\cos^{-1} x) = \frac{2}{\sqrt{5}} \Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \frac{2}{\sqrt{5}}$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow x = \pm\frac{\sqrt{5}}{3}$$

(iii) Prove that the line 2x - 3y = 9 touches the conics $y^2 = -8x$. Also, find the point of contact.

Solution: From the equation of the line, $x = \frac{9+3y}{2}$.

Substituting in the given conic, we have $y^2 = -4(9+3y)$

$$\Rightarrow y^2 + 12y + 36 = 0$$

The above equation has repeated root, showing the line 2x - 3y = 9 touches the given conic.

Clearly, y = -6. Therefore x = -9/2

Thus the point of intersection is $\left(-\frac{9}{2}, -6\right)$

(iv) Using L'Hospital's Rule, evaluate:

$$\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{\cot x}{x}\right)$$

Solution:

$$\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{\cot x}{x}\right) = \lim_{x\to 0}\left(\frac{1 - x\cot x}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{\tan x - x}{x^2\tan x}\right) \quad \frac{0}{0} - form$$

$$= \lim_{x\to 0}\frac{\sec^2 x - 1}{x^2\sec^2 x + 2x\tan x} \quad \frac{0}{0} - form$$

$$= \lim_{x\to 0}\frac{2\sec^2 x\tan x}{2x^2\tan x\sec^2 x + 2\tan x + 4x\sec^2 x} \quad \frac{0}{0} - form$$

$$= \lim_{x\to 0}\frac{2\sec^4 x + 2\sec^4 x\tan x}{6\sec^2 x + 2x^2\sec^4 x + 4x^2\tan^2 x\sec^2 x + 12x\tan x\sec^2 x}$$

$$= \frac{2}{6} = \frac{1}{3}$$

(v) Evaluate: $\int \tan^3 dx$

Solution: $\int \tan^3 dx = \int \tan x\, tan^2\, x\, dx$

$$= \int (\tan x \sec^2 x - \tan x)\, dx$$

$$= \int \tan x \, d(\tan x) - \int \tan x \, dx$$

$$= \frac{\tan^2 x}{2} - \ln|\sec x| + c$$

$$c \text{ is the constant of integration}$$

(vi) Using properties of definite integrals, evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$$

Solution: $Let\ I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\cos x \sin x}\, dx$

$$Using \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx + \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x}\, dx = 0$$

$$\Rightarrow I = 0$$

(vii) The two lines of regression are x+2y-5=0 and 2x+3y-8=0 and the variance of x is 12. Find the variance of y and the coefficient of correlation.

Solution: Assuming the first line to be of y on x, $b_{yx} = -\frac{1}{2}$ therefore $b_{xy} = -\frac{3}{2}$ Clearly $b_{xy} \times b_{yx} = \frac{3}{4} \le 1.$

So our assumption is correct. Also, $r = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$. Find the variance yourself!

(viii) Express $\frac{2+i}{(1+i)(1-2i)}$ in the form $a + ib$. Find its modulus and argument.

Solution: $\frac{2+i}{(1+i)(1-2i)} = \frac{2+i}{3-i} = \frac{5+5i}{10} = \frac{1}{2} + i\frac{1}{2}$

Modulus is $\sqrt{\frac{1}{4}+\frac{1}{4}}=\frac{1}{\sqrt{2}}$

Clearly the complex number lies on the angle bisector in the first quadrant, thus argument is $\frac{\pi}{4}$

(ix) A pair of dice is thrown. What is the probability of getting an even number on the first die or a total of 8?

Solution: Probability of getting an even number on the first dice is $\frac{3\times 6}{36}=\frac{1}{2}$

Probability of getting a total of 8 is $\frac{5}{36}$

{ (2,6),(6,2),(3,5)(5,3),(4,4) }

Probability that both the events occur simultaneously $=$ $\frac{3}{36}=\frac{1}{12}$

Required probability $=\frac{1}{2}+\frac{5}{36}-\frac{1}{12}=\frac{5}{9}$

(x) Solve the differential equation: $x\frac{dy}{dx}+y=3x^2-2$

Solution: Integrating factor is $e^{\int\frac{dx}{x}}=x$

On multiplying with I.F we get

$$\frac{d}{dx}(yx)=3x^2-2$$

Integrating both side,

$$yx=x^3-2x+c$$

c is the constant of integration

Question 2

(a) Using properties of determinants, prove that: [5]

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}=4abc$$

Solution:

$$R_1 \to R_1 - (R_2 + R_3) = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$R_2 \to cR_2 = \frac{1}{c}\begin{vmatrix} 0 & -2c & -2b \\ bc & c(c+a) & bc \\ c & c & a+b \end{vmatrix}$$

$$R_2 \to R_2 - bR_3 = \begin{vmatrix} 0 & -2c & -2b \\ 0 & c(c+a-b) & b(c-a-b) \\ c & c & a+b \end{vmatrix}$$

$$= \frac{1}{c}c(-2bc(c-a-b) + 2bc(c+a-b)$$

$$= 2bc(c+a-b-c+a+b) = 2bc \times 2a = 4abc$$

(b) Solve the following system of linear equations using matrix method:

$3x + y + z = 1, 2x + 2z = 0, 5x + y + 2z = 2$ [5]

Solution: The given system of equation in matrix form is

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ 5 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 0 & 2 \\ 5 & 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 & -1 & 2 \\ 6 & 1 & -4 \\ 2 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\therefore x = 1, y = -1 \; and \; z = -1$$

Question 3

(a) If $\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2}$, prove that $2x^2 + 1 = \sqrt{5}$ [5]

$$\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2} \quad \Rightarrow \tan^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\Rightarrow \tan^{-1} x = \cos^{-1} x$$

$$\Rightarrow \tan^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow\ x^2 = \sqrt{1-x^2}$$

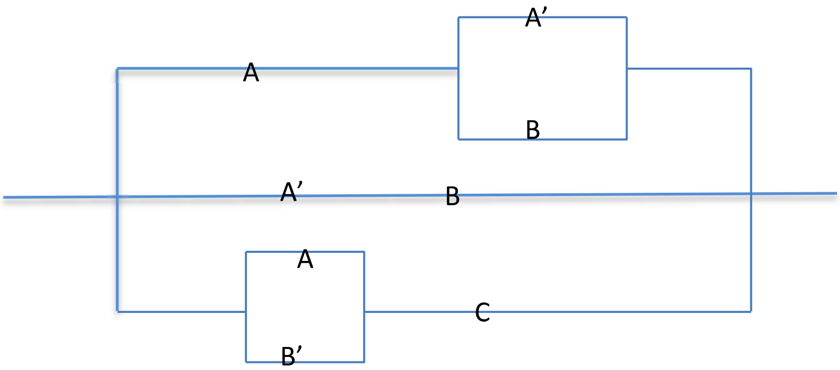
$$put\ t = x^2 \ **$$

$$\Rightarrow\ t = \sqrt{1-t}$$

$$\Rightarrow t^2 + t - 1 = 0, t\ cannot\ be\ negative\ **$$

$$\Rightarrow\ t = \frac{\sqrt{5}-1}{2} \Rightarrow\ 2x^2 + 1 = \sqrt{5}$$

(b) Write the Boolean function corresponding to the switching circuit given below: [5]

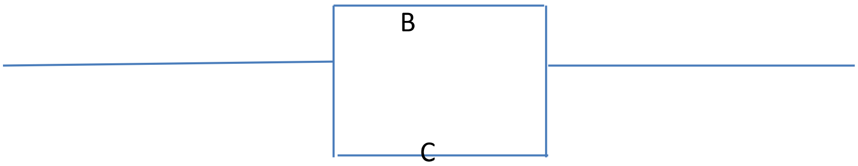


A,B and C represent switches in 'on' position and A', B' and C' represent them in 'off' position. Using Boolean algebra, simplify the function and construct an equivalent switching circuit.

Solution: The given circuit represent the function

$$A.(A' + B) + A'.B + (A + B').C$$
$$= A.B + A'.B + A.C + B'.C$$
$$= B + B'.C + A.C$$
$$= (B + B').(B + C) + A.C$$
$$= B + C + A.C$$
$$= B + C$$

Simplified Circuit



Question 4

(a) Verify the conditions of Rolle's Theorem for the following function:

$f(x) = \ln(x^2 + 2) - \ln 3 \ on \ [-1,1]$ [5]

Find a point in the interval, where the tangent to the curve is parallel to x-axis.

Solution: $x^2 + 2$ is positive on the given interval further since $\ln x \ and \ x^2 + 2$ are continuous and differentiable throughout in their domain $(R^+ \ and \ R)$ their composition will be continuous and differentiable wherever they are defined.

Also, $f(1) = f(-1) = 0$

Thus all the conditions of the Rolle's Theorem is verified which in turn implies there exists at least one real value in (-1,1) such that at that point the derivative of f vanishes. And at that point the tangent to f will be parallel to x-axis.

$f'(x) = 0 \Rightarrow \frac{2x}{x^2+2} = 0 \Rightarrow x = 0$

Thus the required point is $\left(0, \ln\frac{2}{3}\right)$

(b) Find the equation of the standard ellipse, taking its axes as the coordinate axes, whose minor axis is equal to the distance between the foci and whose length of latus rectum is 10. Also, find its eccentricity. [5]

Solution: With the symbols having their usual meaning we have $2b = 2ae \ and \ \frac{2b^2}{a} = 10 \Rightarrow b = ae \ and \ b^2 = 5a$

Squaring the first gives, $b^2 = a^2e^2 \Rightarrow 5a = a^2e^2 \Rightarrow \frac{5}{a} = e^2$

Note that ‘a’ is a positive quantity.

Again, $e^2b^2 = a^2 - b^2 \Rightarrow \frac{5}{a} \times 5a = a^2 - 5a$

$$\Rightarrow a^2 - 5a - 25 = 0 \Rightarrow a = \frac{5}{2} + \frac{5\sqrt{5}}{2}$$

$$\Rightarrow a^2 = \frac{25}{4}\left(1+\sqrt{5}\right)^2$$

$$\therefore b^2 = \frac{25}{2}\left(1+\sqrt{5}\right)$$

$$and\ e = \sqrt{\frac{5}{a}} = \sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}$$

Equation of the ellipse is $\frac{x^2}{\frac{25}{4}\left(1+\sqrt{5}\right)^2} + \frac{y^2}{\frac{25}{2}\left(1+\sqrt{5}\right)} = 1$

Question 5

(a) If $\ln y = \tan^{-1} x$, prove that:

$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ [5]

Solution: Differentiating the given equation w.r.t ‘x’

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2} \Rightarrow (1+x^2)\frac{dy}{dx} = y$$

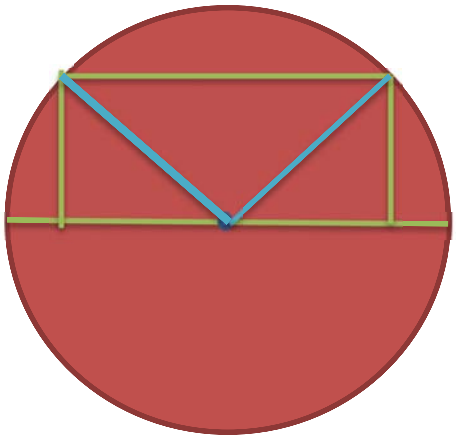
Differentiating again w.r.t ‘x’ we get

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}\,2x = \frac{dy}{dx}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

(b) A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle to get maximum area. Also, find the maximum area. [5]

Solution:



Join the centre of the circle with two of its vertices as shown in the diagram. By R-H-S criteria of congruency both the triangles are congruent. (radius is the hypotenuse, two sides of rectangle are orthogonal, opposite sides of rectangle are equal).

Therefore the side on the diameter is symmetrical about the centre of the circle, let its length be 2x and that of perpendicular to diameter be y units.

Area of the rectangle $A = 2xy \Rightarrow A^2 = 4x^2y^2$

Again, $r^2 = x^2 + y^2$

Therefore, $A^2 = 4x^2(r^2 - x^2) = 4(r^2x^2 - x^4)$

Note both $A\ and\ A^2$ have maximum or minimum values at the same point since A is positive.

At max. or min. values of $A^2, \frac{dA^2}{dx} = 0$

$$\Rightarrow 4(r^2 2x - 4x^3) = 0 \ \Rightarrow x = \frac{r}{\sqrt{2}}$$

$$\frac{d^2A^2}{dx^2}\Big|_{x=\frac{r}{\sqrt{2}}} = 8(r^2 - 6x^2)_{x=\frac{r}{\sqrt{2}}} = 8(r^2 - 3r^2) < 0$$

Therefore $A^2\ is\ maximum\ at\ x = \frac{r}{\sqrt{2}}$

Now, $r^2 = x^2 + y^2 \Rightarrow y = \frac{r}{\sqrt{2}}$

Maximum area is $A = 2xy = r^2\ sq.units$

Question 6

a) Evaluate: $\int \frac{\sin x+\cos x}{\sqrt{9+16\sin 2x}}dx$

Solution: $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)\,dx = dt$

$$also, 1 - sin2x = t^2 \Rightarrow 1 - t^2 = \sin 2x$$

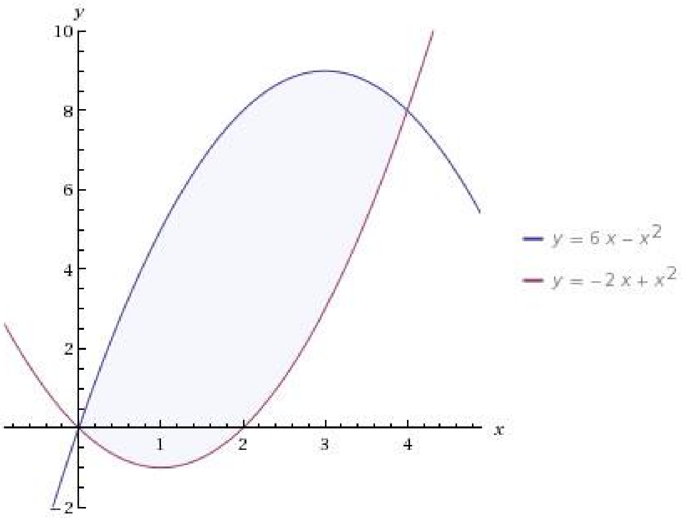
$$\therefore \int \frac{\sin x + \cos x}{\sqrt{9 + 16\sin 2x}}dx = \int \frac{dt}{\sqrt{9 + 16(1 - t^2)}}$$

$$= \int \frac{dt}{\sqrt{25 - 16t^2}} = \frac{1}{4}\int \frac{dt}{\sqrt{\left(\frac{5}{4}\right)^2 - t^2}} = \frac{1}{4}\sin^{-1}\frac{4t}{5} + c$$

$$= \frac{1}{4}\sin^{-1}\left(\frac{4(\sin x - \cos x)}{5}\right) + c$$

b) Find the area of the region bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.

Solution:



Required Area is $= \int_0^4 (6x - x^2 + 2x - x^2)dx =$ $\int_0^4 (8x - 2x^2)\,dx = \frac{64}{3}$ $unit\ square$

Question 7

a) Calculate Karl Pearson's coefficient of correlation between x and y for the following data and interpret the result: [5]
(1,6), (2,5), (3,7), (4,9), (5,8), (6,10), (7,11), (8,13), (9,12)
Solution:

X	Y	XY	X^2	Y^2
1	6	6	1	36
2	5	10	4	25
3	7	21	9	49
4	9	36	16	81
5	8	40	25	64
6	10	60	36	100
7	11	77	49	121
8	13	104	64	169
9	12	108	81	144
45	81	462	285	789

Karl Pearson's coefficient of correlation =

$$\frac{9\times462-45\times81}{\sqrt{(9\times285-(45)^2)(9\times789-(81)^2)}} = 0.95$$

Interpret yourself!

b) The marks obtained by 10 candidates in English and Mathematics are given below: [5]

Marks in English	20	13	18	21	11	12	17	14	19	15
Marks in Mathematics	17	12	23	25	14	8	19	21	22	19

Estimate the probable score for Mathematics if the marks obtained in English are 24.
Solution:
Let 'x' represent the score obtained in English while 'y' represent the score obtained in Mathematics.
To obtain the probable score of Mathematics based on the score of English we have to find the regression line of 'y' on 'x'.

The regression line is $y = -0.182x + 1.136x$
Now obtain the desired result!

Question 8

a) A committee of 4 persons has to be chosen from 8 boys and 6 girls, consisting of at least one girl. Find the probability that the committee consists of more girls than boys. [5]

Solution: Total number of ways in which the committee can be formed is (one girl being already selected)

$^5C_0 \times {}^8C_3 + {}^5C_1 \times {}^8C_2 + {}^5C_2 \times {}^8C_1 + {}^5C_3 \times {}^8C_0 = 286$

Alternately $^{13}C_3 = 286$

Number of committee which consists of more girls than boys is $^6C_3 \times {}^8C_1 + {}^6C_4 \times {}^8C_0 = 175$

Required probability is $\frac{175}{286}$ **

b) An urn contains 10 white and 3 black balls while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second run and then a ball is drawn from the second urn. Find the probability that the ball drawn from the second urn is a white ball. [5]

Solution:

CASE I: Two white balls are transferred

Probability of getting a white ball from the second urn is

$$\frac{^{10}C_2}{^{13}C_2} \times \frac{5}{10} = \frac{15}{52}$$

CASE II: Two black balls are transferred

Probability of getting a white ball from the second urn is

$$\frac{^{3}C_2}{^{13}C_2} \times \frac{3}{10} = \frac{3}{260}$$

CASE III: One black and one white ball transferred to the second urn

Probability of getting a white ball from the second urn is

$$\frac{10 \times 3}{^{13}C_2} \times \frac{4}{10} = \frac{2}{13}$$

Clearly the required probability is $= \frac{15}{52} + \frac{3}{260} + \frac{2}{13} = \frac{59}{130}$

Question 9

Find the locus of a complex number, $z = x + iy$, satisfying the relation $\left|\frac{z-3i}{z+3i}\right| \leq \sqrt{2}$. Illustrate the locus of z in the Argand plane. [5]

Solution: $\left|\frac{z-3i}{z+3i}\right| \leq \sqrt{2} \Rightarrow \left|\frac{z-3i}{z+3i}\right|^2 \leq 2$

$$\Rightarrow |z-3i|^2 \leq 2|z+3i|^2$$

$$Let\ z = x + iy, the\ above\ inequation\ reduces\ to$$

$$x^2 + (y-3)^2 \leq 2x^2 + 2(y+3)^2$$

$$\Rightarrow x^2 + y^2 + 18y + 9 \geq 0$$

$which\ represent\ the\ exterior\ of\ a\ circle\ with\ centre\ at\ (0, -9)$

a) Solve the following differential equation:

$x^2dy + (xy + y^2)dx = 0, when\ x = 1\ and\ y = 1$ [5]

Solution: $x^2dy + (xy + y^2)dx = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{xy + y^2}{x^2}$$

$$Put\ y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = -\frac{x^2(v + v^2)}{x^2} = -v - v^2$$

$$\Rightarrow \frac{dv}{v^2+2v}+\frac{dx}{x}=0$$

$$\Rightarrow \frac{1}{2}(\ln v-\ln(v+2))+\ln\ x=c$$

where c is the constant of integration.

$$\frac{1}{2}\ln\frac{\frac{y}{x}}{\frac{y}{x}+2}+\ln x=c\ \Rightarrow \frac{1}{2}\ln\frac{y}{y+2x}+\ln x=c$$

From the initial conditions,

$$c=\frac{1}{2}\ln\frac{1}{3}=-\frac{1}{2}\ln 3$$

$$\therefore \frac{1}{2}\ln\frac{y}{y+2x}+\ln x=-\frac{1}{2}ln3$$

$$other\ forms\ are\ possible$$

SECTION B (20 MARKS)

Question 10

a) For any three vectors $\vec{a},\vec{b},\vec{c}$, show that $\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}$ are coplanar. [5]

Solution: We compute the scalar triple product of the vectors $\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}$

$$(\vec{a}-\vec{b}).\left((\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\right)$$

$$=(\vec{a}-\vec{b}).(\vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a})$$

$$=[\vec{a}\vec{b}\vec{c}]+0+0-0-0-[\vec{b}\vec{c}\vec{a}]=0$$

which verifies the desired result.

b) Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}=3i+2j+2k\ and\ \vec{b}=i+2j-2k$ [5]

Solution: From the given conditions, the required unit vector is parallel to $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$

$$(\vec{a}+\vec{b})=4i+4j+0k,\vec{a}-\vec{b}=2i+0j+4k$$

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=16i-16j-8k$$

Thus the required unit vector is $=\pm\frac{16i-16j-8k}{24}$

Question 11

a) Find the image of the point (2,-1,5) in the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$ [5]

Solution: Let the co-ordinate of the foot of the perpendicular from the point (2,-1,5) be $(10t+11,-4t-2,-11t-8)$ where t is parameter.

Line joining the above two points is perpendicular to the given line

$\therefore 10(10t+11-2)-4(-4t-2+1)-11(-11t-8-5)=0$

$\Rightarrow\ t=-1$

Thus the foot of the perpendicular is $(1,2,3)$

Using mid-point formula co-ordinate of the image is

$(1\times 2-2,2\times 2+2,3\times 2-5)=(0,6,1)$

b) Find the Cartesian equation of the plane, passing through the line of intersection of the planes: $\vec{r}.(2i+3j-4k)+5=0\ and\ \vec{r}.(i-5j+7k)+2=0$ and intersecting y-axis at (0,3,0). [5]

Solution: Equation of the plane through the intersection of the given two planes is $\vec{r}.(2i+3j-4k)+5+t(\vec{r}.(i-5j+7k)+2)=0$ where t is a parameter.

$$\Rightarrow (2+t)x+(3-5t)y+(-4+7t)z+5+2t=0$$

Since it passes through (0,3,0) we have

$$9-15t+5+2t=0\ \Rightarrow 14=13t\ \Rightarrow\ t=\frac{14}{13}$$

Therefore the equation of the plane is

$$\left(2+\frac{14}{13}\right)x+\left(3-5\frac{14}{13}\right)y+\left(-4+7\frac{14}{13}\right)z+5+2\frac{14}{13}=0$$

$$\Rightarrow\ 40x-31y+46z+93=0$$

Question 12

a) In an automobile factory, certain parts are to be fixed into the chassis in a section before it moves into another section. On a given day, one of the three persons A,B and C carries out this task. A has 45% chance, B has 35% chance and C has 20% chance of doing the task. The probability that A,B and C will take more than the allotted time is $\frac{1}{6},\frac{1}{10}, and\, \frac{1}{20}$ respectively. If it is found that the time taken is more than the allotted time, what is the probability that A has done the task? [5]

Solution: Let X be the event "time taken is more than the allotted time". Further let $E_1, E_2\ and\ E_3$ be the events that the task was carried out by A,B and C respectively. Now,

$$P\left(\frac{E_1}{X}\right)=\frac{P\left(\frac{X}{E_1}\right)P(E_1)}{P\left(\frac{X}{E_1}\right)P(E_1)+P\left(\frac{X}{E_2}\right)P(E_2)+P\left(\frac{X}{E_3}\right)P(E_3)}$$

$$=\frac{\frac{1}{6}\times\frac{45}{100}}{\frac{1}{6}\times\frac{45}{100}+\frac{1}{10}\times\frac{35}{100}+\frac{1}{20}\times\frac{20}{100}}=\frac{5}{8}$$

b) The difference between mean and variance of a binomial distribution is 1 and the difference of their squares is 11. Find the distribution. [5]

Solution: With symbols having their usual meanings, we have

$p+q=1, np-npq=1\ and\ n^2p^2-n^2p^2q^2=11$

Solving we get, $n=36, p=\frac{1}{6}$

SECTION C (20 Marks)

Question 13

a) A man borrows ₹ 20,000 at 12% per annum, compounded semi-annually and agrees to pay it in 10 equal semi-annual instalments. Find the value of each instalment, if the first payment is due at the end of two years. [5]

b) A company manufactures two types of products A and B. Each unit of A requires 3 grams of nickel and 1 gram of chromium, while each unit of B requires 1 gram of nickel and 2 grams of chromium. The firm can produce 9 grams of nickel and 8 grams of chromium. The profit is ₹ 40 on each unit of product of type A and ₹ 50 on each unit of type B. How many units of each type should the company manufacture so as to earn maximum profit? Use linear programming to find the solution. [5]

Solution: Let x and y units of products A and B is being manufactured.

Profit function is $z = 40x + 50y, x, y \geq 0$

Product/Minerals	Nickel	Chromium
A (x units)	3x	x
B (y units)	y	2y

According to the problem,

$3x + y \leq 9$ and $x + 2y \leq 8$ (Production constraint)

The L.P.P is $Maximize\ \ z = 40x + 50y$

$$\text{Subjected to}\quad \begin{aligned} x, y &\geq 0 \\ 3x + y &\leq 9 \\ x + 2y &\leq 8 \end{aligned}$$

Now solve!

Question 14

a) The demand function is $x = \frac{24-2p}{3}$ where x is the number of units demanded and p is the price per unit. Find: i) The revenue function R in terms of p. ii) The price and the number of units demanded for which the revenue is maximum. [5]

b) A bill ₹ 1,800 drawn on 10th September, 2010 at 6 months was discounted for ₹ 1,782 at a bank. If the rate of interest was 5% per annum, on what date was the bill discounted? [5]

Question 15

a) The index number by the method of aggregates for the year 2010, taking 2000 as the base year, was found to be 116. If the sum of the prices in the year 2000 is ₹300, find the values of x and y in the data given below: [5]

Commodity	A	B	C	D	E	F
Price in the year 2000 (₹)	50	X	30	70	116	20
Price in the year 2010 (₹)	60	24	Y	80	120	28

b) From the details given below, calculate the five yearly moving averages of the number of students who have studies in a school. Also, plot these and original data on the same graph paper. [5]

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Number of Students	332	317	357	392	402	405	410	427	405	438

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