MATHEMATICS

(Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper.*
They must NOT start writing during this time)

Section A - Answer **Question 1** *(compulsory)* and **five** other questions.

Section B and Section C - Answer **two** questions from **either** Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to,

the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A

Question 1

(i) If the matrix $A = \begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is a singular matrix, find the value of x. -3 [3]

(ii) Solve: $\cos^{-1}[\sin(\cos^{-1}x)] = \pi/3$ $\sqrt{3}/2$ [3]

(iii) Show that the line $y = x + \sqrt{7}$ touches the hyperbola $9x^2 - 16y^2 = 144$. [3]

(iv) Evaluate: $\displaystyle \lim_{x \to \pi/2}\left[x\tan x - \frac{\pi}{2}\sec x\right]$ $-1/2$ [3]

(v) Evaluate: $\displaystyle \int \frac{x}{(x+1)^2}dx$ [3]

(vi) Evaluate: $\displaystyle \int_{-3}^{3}|x+2|\,dx$ [3]

(vii) A fair die is thrown once. What is the probability that either an even number or a number greater than three will turn up? $\times 2$ [3]

(viii) If the regression equation of x on y is given by $mx - y + 10 = 0$ and the equation of y on x is given by $-2x + 5y = 14$, determine the value of 'm' if the coefficient of correlation between x and y is $\dfrac{1}{\sqrt{10}}$. 4 [3]

This Paper consists of 5 printed pages and 1 blank page.

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Turn over

(ix) If $1, \omega, \omega^2$ are the three cube roots of unity, then simplify: [3]
$(3 + 5\omega + 3\omega^2)^2 (1 + 2\omega + \omega^2)$

(x) Solve the differential equation: $\cosec^3 x \, dy - \cosec y \, dx = 0$ [3]

Question 2

(a) By using properties of determinants, prove that the determinant [5]
$$\begin{vmatrix} a & \sin x & \cos x \\ -\sin x & -a & 1 \\ \cos x & 1 & a \end{vmatrix}$$ is independent of x.

(b) Using matrix method, solve the following equations: [5]
$5x + 3y + z = 16$
$2x + y + 3z = 19$
$x + 2y + 4z = 25$

Question 3

(a) Using Rolle's theorem, find a point on the curve $y = \sin x + \cos x - 1$, $x \in [0, \frac{\pi}{2}]$ where [5]
the tangent is parallel to the x-axis.

(b) Find the equation of the parabola whose focus is $(-1, -2)$ and the equation of the [5]
directrix is given by $4x - 3y + 2 = 0$. Also find the equation of the axis.

Question 4

(a) Prove that: [5]
$$\sin \left[2 \,\mathrm{Tan}^{-1} \frac{3}{5} - \mathrm{Sin}^{-1} \frac{7}{25} \right] = \frac{304}{425}$$

(b) x, y and z represent three switches in an 'ON' position and x', y' and z' represent the same [5]
three switches in an 'OFF' position. Construct a switching circuit representing the
polynomial $(x + y)(x' + z) + y(y' + z')$.

Using the laws of Boolean Algebra, show that the above polynomial is equivalent
to $xz + y$ and construct an equivalent switching circuit.

Question 5

(a) Using a suitable substitution, find the derivative of $\mathrm{Tan}^{-1} \sqrt{\dfrac{a-x}{a+x}}$ with respect to x. [5]

(b) A closed right circular cylinder has volume $\dfrac{539}{2}$ cubic units. Find the radius and the [5]
height of the cylinder so that the total surface area is minimum.

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Question 6

(a) Evaluate: $\displaystyle\int \frac{2\sin 2\theta - \cos\theta}{6 - \cos^2\theta - 4\sin\theta}\, d\theta$ **[5]**

(b) Draw a rough sketch of the curve $y = x^2 - 5x + 6$ and find the area bounded by the curve **[5]** and the x–axis.
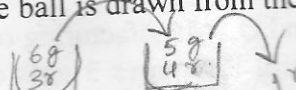
Question 7

(a) Find the equations of the two lines of regression for the following observations: **[5]**

$(3, 6), (4, 5), (5, 4), (6, 3), (7, 2)$

Find an estimate of y for $x = 2.5$

(b) Calculate Spearman's coefficient of rank correlation from the following data and interpret **[5]** the result:

x	16	19	22	28	25	31	37	40	43	49
y	25	25	27	31	27	33	35	41	45	41

Question 8

(a) Akhil and Vijay appear for an interview for two vacancies. The probability of Akhil's **[5]** selection is $\dfrac{1}{4}$ and Vijay's selection is $\dfrac{2}{3}$. Find the probability that only one of them will be selected.

(b) There are two bags. One bag contains six green and three red balls. The second bag **[5]** contains five green and four red balls. One ball is transferred from the first bag to the second bag. Then one ball is drawn from the second bag. Find the probability that it is a red ball.

Question 9

(a) Solve the differential equation: $(y + \log x)\, dx - x\, dy = 0$, given that $y = 0$, when $x = 1$. **[5]**

(b) Find the locus of a complex number $z = x + iy$, satisfying the relation $|3z - 4i| \le |3z + 2|$. **[5]** Illustrate the locus in the Argand plane.

Turn over

Question 14

(a) A bill of exchange for Rs. 722 was drawn on the 3rd April, 2009, payable three months after date. It was discounted on 15th April, 2009 at 4.75% per annum. What was the discounted value of the bill? **[5]**

(b) The average cost function AC for a commodity is given by $AC = x + 5 + \dfrac{36}{x}$ in terms of output x. Find the: **[5]**

    (i) Total cost and the marginal cost as the functions of x.

    (ii) Output for which AC increases.

Question 15

(a) The index number for the following data, for the year 2008, taking 2004 as base year was found to be 116. The simple aggregate method was used for calculation. Find the numerical values of x and y if the sum of the prices in the year 2008 is Rs. 203. **[5]**
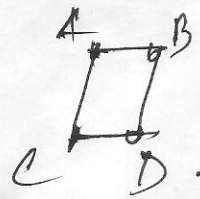
Commodity	Price in Rs. in the year 2004	Price in Rs. in the year 2008
A	20	25
B	10	30
C	30	15
D	25	45
E	x	35
F	50	y

(b) Consider the following data:

[5]

Dates in the month of April	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Number of units sold	2	5	0	12	13	25	45	13	31	18	11	2	3	1

Calculate three days moving averages and display these and the original figures on the same graph.

SECTION B

Question 10

(a) Find the value of λ for which the four points with position vectors $2\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - 4\hat{k}$, $3\hat{i} + \lambda \hat{j} + 8\hat{k}$ and $-4\hat{i} + 3\hat{j} + 4\hat{k}$ are coplanar. [5]

(b) In any $\triangle ABC$, prove by vector method that $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$. [5]

Question 11

(a) Find the shortest distance between the lines whose vector equations are: [5]

$$\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and } \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} + 2\hat{j} - 4\hat{k}).$$

(b) Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $3z - y = 0$ and perpendicular to the plane $3x + 4y - 2z + 6 = 0$. [5]

Question 12

(a) A factory has three machines A, B and C producing 1,500, 2,500 and 3,000 bulbs per day, respectively. Machine A produces 1.5% defective bulbs, machine B produces 2% defective bulbs and machine C produces 2.5% defective bulbs. At the end of the day, a bulb is drawn at random and is found to be defective. What is the probability that this defective bulb has been produced by machine B? [5]

(b) Five bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, find the probability distribution of the number of good eggs drawn. [5]

SECTION C

Question 13

(a) A company produces two types of items, P and Q. Manufacturing of both items requires the metals gold and copper. Each unit of item P requires 3 gms of gold and 1 gm of copper while that of item Q requires 1 gm of gold and 2 gms of copper. The company has 9 gms of gold and 8 gms of copper in its store. If each unit of item P makes a profit of Rs.50 and each unit of item Q makes a profit of Rs.60, determine the number of units of each item that the company should produce to maximize profit. What is the maximum profit? [5]

(b) At the beginning of each quarter, a sum of Rs. 1,500 is deposited into a savings account that pays 12% per annum compounded quarterly. Find the amount in the account at the end of four years. [5]