

ISC 2009
BUSINESS MATHEMATICS
Part I

Answer all questions

i) Find the fourth term in the expansion $(x^3/2-2/x^2)^9$

ii) Use the principle of induction to prove that:

$$1/2+1/4+1/8+\dots+1/2^n=1-1/2^n \text{ for all } n \in \mathbb{N}$$

iii) How many different seven letter words can be formed with the letters of the word 'CAPTAIN'? How many of these words begin with C and end with N? In how many of these words, 'C' and 'T' occur together.

iv) Find the equation of the hyperbola with directrices as $x=\pm 4$ and foci as $(\pm 6,0)$.

v) Differentiate $\sin(\log(ax^2 + b^2))$ with respect to x.

vi) Integrate $\int \sin^{\frac{2}{3}} x \cos^3 x \, dx$

vii) Find the value of x, for which the matrix A is singular, where: $A = \begin{pmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{pmatrix}$

viii) Find the mean deviation about the mean for the given data:
2,4,5,7,8,10,12,17,19,26

ix) 7 boys and 3 girls sit in a row at random. Find the probability that no two girls sit together.

x) Using Cramer's rule solve the following system of equations:
 $x-2y-4=0$
 $-3x+5y+7=0$

PART II
SECTION A

Answer any four questions.

Question 2

(a) If the coefficients of x^9 and x^{10} in the binomial expansion of $(3 + \frac{x}{2})^n$ are equal, find n.

(b) For a cricket team, 6 students from class XI and 8 students from class XII have come from selection. In how many ways can a team of 11 students be

selected taking at least two students from class XI and at least one student from class XII?

Question

(a) prove, using the properties of determinants that:

$$\begin{vmatrix} y+z & x+y & x \\ z+x & y+z & y \\ x+y & z+x & z \end{vmatrix}$$

$$= x^3 + y^3 + z^3 - 3xyz$$

$$= x^3 + y^3 + z^3 - 3xyz$$

is equal to $x^3 + y^3 + z^3 - 3xyz$

(b) Find the equation of the ellipse with the vertices $(\pm 5, 0)$ and foci $(\pm 4, 0)$.

Question 4

(a) Solve the following system of equations using the matrix method:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

Question 5

(a) Differentiate $y = \arctan\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

(b) if $y = \log x = x - y$,

prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Question 6

Integrate the following

(a) $\int x^5 \cos(x^3) dx$

(b) $\int \frac{(x+\log x)^2(x+1)}{x} dx$

Question 7

(a) An open tank with a square base of side 'x' metres and vertical height 'h' metres is to be constructed so as to contain 'c' cubic metres of water. Show that the expenses on lining the inside of the tank with lead would be least if $h = \frac{x}{2}$

(b) Find $\frac{dy}{dx}$, if $\tan(x+y) + \tan(x-y) = 1$.

SECTION B

Answer any two questions.

Question 8

Evaluate the following

(a) $\int_0^1 \frac{e^x}{1+e^{2x}} dx$

(b) $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

Question 9

- (a) Find the area of the region bounded by the curve $x^2=4y$ and the straight line $x=4y-2$.
- (b) Find the volume of the solid generated by revolving about the x-axis, the area bounded by the parabola $y^2=12x$ and its latus rectum.

Question 10

Solve the following:

- (a) $\frac{dy}{dx}=e^{x-y}+x^2 e^{-y}$
- (b) $\sin x \frac{dy}{dx}+y \cos x=2\sin^2 x \cos x$

SECTION C

Answer any two questions.

Question 11

- (a) Calculate the coefficient of variation for the following data:

Size of the item: 10 11 12 13 14 15 16

Frequency: 2 7 11 15 10 4 1

- (b) The number of traffic offences committed in a certain city during a certain period is given below:

Year : 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984

No. of traffic offences: 74 56 48 69 83 52 49 81 94 85

Calculate the 3-yearly moving averages.

Question 12

- (a) A firm took an examination of eight applicants for a clerical post. The marks obtained by the applicants in Accounts and Statistics are given below. Compute the Spearman's rank correlation coefficient and comment on its value.

Marks in Accounts: 15 20 28 12 40 60 20 80

marks in Statistics: 40 30 50 30 20 10 30 60

- (b) Out of the following two regression lines, find the line of regression of x on y: $2x+3y=7$ and $5x+4y=9$.

Question 13

- (a) Six cards are drawn at random from a pack of 52 cards. What is the probability that three will be red and three will be black?

- (b) In a class, 5 percentage of the boys and 10 percentage of the girls have an IQ more than 150. In this class, 60 percentage of the students are boys. If a student is selected at random and found to have an IQ more than 150, find the probability that the student is a boy.