Solved IIT JEE Mathematics 2011 Paper-1
SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. A straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is

(A) $\sqrt{3}x+y+2-3\sqrt{3}=0$ (B) $-\sqrt{3}x+y+2+3\sqrt{3}=0$

(C) $\sqrt{3}y-x+3+2\sqrt{3}=0$ (D) $\sqrt{3}y+x-3+2\sqrt{3}=0$

Equation of the required line is $y=mx-3m-2$, where m is the slope. According to the problem $\tan 60^0=\left|\frac{m+\sqrt{3}}{1-m\sqrt{3}}\right|$, solving we get $m=\sqrt{3}, m\neq 0$ as the required line intersect the $x-axis$. Hence Ans: (B).

2. Let (x_0, y_0) be solution of the following equations $(2x)^{\ln 2}=(3y)^{\ln 3}$ and $3^{\ln x}=2^{\ln y}$. Then x_0 is

(A) 1/6 (B) 1/3 (C) 1/2 (D) 6

The first equation can be written as

$\ln 2\ln(2x)=\ln 3\ln(3y)$ *or* $\ln 2(\ln 2+\ln x)=\ln 3(\ln 3+\ln y)$ and the second one as $\ln x\ln 3=\ln y\ln 2$ Therefore $\ln y=\frac{\ln x\ln 3}{\ln 2}$ substituting in above and solving we get $x=\frac{1}{2}$. Ans: (C).

3. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}\frac{x\sin x^2}{\sin x^2+\sin(\ln 6-x^2)}dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\ln\frac{3}{2}$ (D) $\frac{1}{6}\ln\frac{3}{2}$

let $I=\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}\frac{x\sin x^2}{\sin x^2+\sin(\ln 6-x^2)}dx$ and put $\ln 6-x^2=t^2$, this will give

$I=-\int_{\sqrt{\ln 3}}^{\sqrt{\ln 2}}\frac{t\sin(\ln 6-t^2)}{\sin(\ln 6-t^2)+\sin t^2}dt=\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}\frac{t\sin(\ln 6-t^2)}{\sin(\ln 6-t^2)+\sin t^2}dt$, thus $I+I=\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}tdt$

hence I=$\frac{1}{4}\ln\frac{3}{2}$ Ans (A).

4. Let $a=\hat{i}+\hat{j}+\hat{k}$, $b=\hat{i}-\hat{j}+\hat{k}$ and $c=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector v in the plane of a and b, whose projection on v is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$ (C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

Ans: (C).

5. Let P = $\left\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\right\}$ and Q = $\left\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\right\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \phi$ (B) Q $\not\subset$ P

(C) P $\not\subset$ Q (D) P = Q

$$P = \left\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\right\} = \left\{\theta : \sin\theta = \cos\theta + \sqrt{2}\cos\theta\right\}$$
$$= \left\{\theta : \sin\theta = (1+\sqrt{2})\cos\theta\right\} = \left\{\theta : \tan\theta = (1+\sqrt{2})\right\}$$
$$Q = \left\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\right\} = \left\{\theta : \cos\theta = \sqrt{2}\sin\theta - \sin\theta\right\}$$
$$= \left\{\theta : \cos\theta = (\sqrt{2}-1)\sin\theta\right\} = \left\{\theta : \tan\theta = \frac{1}{(\sqrt{2}-1)}\right\} = \left\{\theta : \tan\theta = \sqrt{2}+1\right\}$$ Thus P=Q

Ans: (D).

6. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(A) 1 (B) 2 (C) 3 (D) 4

$a_{n+1} = \alpha^{n+1} - \beta^{n+1}$

or, $a_{n+1} = \alpha^n\alpha - \beta^n\alpha + \beta^n\alpha + \alpha^n\beta - \alpha^n\beta - \beta^n\beta$

or, $a_{n+1} = (\alpha^n - \beta^n)\alpha + \beta^n\alpha + (\alpha^n - \beta^n)\beta - \alpha^n\beta$

or, $a_{n+1} = (\alpha+\beta)(\alpha^n - \beta^n) - \alpha\beta(\alpha^{n-1} - \beta^{n-1})$

or, $a_{n+1} = (\alpha+\beta)a_n - \alpha\beta a_{n-1}$

or, $a_{n+1} = 6a_n + 2a_{n-1}$

therefore, $a_{10} = 6a_9 + 2a_8 - 2a_8 = 6a_9$

it follows required value is 3 Ans (C).

7. Let the straight line $x = b$ divides the area enclosed by $y = (1-x)^2, y = 0$ and $x = 0$ into two parts $R_1\left(0 \leq x \leq b\right)$ and $R_2\left(b \leq x \leq 1\right)$ such $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) ¾ (B) ½ (C)1/3 (D) ¼

$$\int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2 = \frac{1}{4} \Rightarrow b = \frac{1}{2}$$ Ans (B).

SECTION – II (Total Marks : 16)

(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out ofwhich **ONE or MORE** may be correct.

8. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2-3y^2=3$

Eccentricity of the ellipse $e=\sqrt{3}/2$ and focus is $(\pm\sqrt{3},0)$. Eccentricity of the hyperbola $e_h=\sqrt{1+\frac{b^2}{a^2}}$ According to the problem $\sqrt{1+\frac{b^2}{a^2}}=\frac{2}{\sqrt{3}}$ which gives $3b^2=a^2$, again the hyperbola passes through $(\pm\sqrt{3},0)$ therefore $a^2=3$ hence $b^2=1$ thus the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{1}=1 \Rightarrow x^2-3y^2=3$, clearly the focus is (2,0) Ans: (B) and (D).

9. Let f: R $\rightarrow$ R be a function such that f(x + y) = f(x) + f(y), $\forall x, y \in R$. If f(x) is differentiable at x = 0, then

(A) f(x) is differentiable only in a finite interval containing zero

(B) f(x) is continuous $\forall x \in R$

(C) f'(x) is constant $\forall x \in R$

(D) f(x) is differentiable except at finitely many points

Ans: (B) and (C) taking x=y=0 we see that f(0)=0. assume f'(0)=t, t is real

Now $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(h)}{h}=f'(0)=t$

Therefore $f(x)=tx+c$, since f(0)=0 it follows that $f(x)=tx$, it now follows that the function is everywhere continuous and differentiable.

10. Let M and N be two 3 $\times$3 non-singular skew symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

The problem is wrong as any skew-symmetric matrix of odd order is singular!

11. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

Ans: (A) and (D) Try yourself!

SECTION-III (Total Marls : 15)
(Paragraph Type)
This section contains 2 paragraphs. Based upon one of paragraphs 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 12 to 16

Let a, b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad ....E$$

12. If the point P(a, b, c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is
(A) 0 (B) 12 (C) 7 (D) 6

Equation E is a system of homogeneous equation with infinite solutions

$$a + 8b + 7c = 0$$
$$9a + 2b + 3b = 0$$
$$7a + 7b + 7c = 0$$

Solving by cross-multiplication we get $\frac{a}{10} = \frac{-b}{-60} = \frac{c}{-70}$ which gives b=6a and c=-7a
According to the problems 2a+b+c=1 which gives a=1, thus b=6 and c=-6. therefore 7a+b+c=7+6-7=6 Ans: (D)

13. Let ϖ be a solution of $x^3 - 1 = 0$ with $\mathrm{Im}(\varpi) > 0$. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\varpi^a} + \frac{1}{\varpi^b} + \frac{3}{\varpi^c}$ is equal to
(A) –2 (B) 2 (C) 3 (D) – 3
When a=2 we see that b=12 and c=-14.
Therefore $\frac{3}{\varpi^a} + \frac{1}{\varpi^b} + \frac{3}{\varpi^c} = \frac{3}{\varpi^2} + \frac{1}{\varpi^{12}} + \frac{3}{\varpi^{-14}}$
$=$

14. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right)^n$
(A) 6 (B) 7 (C) 6/7 (D) ∞

Ans: (B) 7
With b=6, we have a=1 and c=-7. therefore the quadratic equation becomes $x^2 + 6x - 7 = 0$, it follows that $\alpha + \beta$ =-6 and $\alpha\beta$ =-7.

Now $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right)^n = \sum_{n=0}^{\infty} \left(\frac{\beta + \alpha}{\alpha\beta} \right)^n = \sum_{n=0}^{\infty} \left(\frac{6}{7} \right)^n = \frac{1}{1 - \frac{6}{7}} = 7$.

Paragraph for Question Nos. 15 to 16
Let U1 and U2 be two urns such that U1 contains 3 white and 2 red balls, and U2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U1 and put into U2. However, if tail appears then 2 balls are drawn at random from U1 and put into U2. Now 1 ball is drawn at random from U2.

15. The probability of the drawn ball from U2 being white is
(A) 13/30 (B) 23/30 (C) 19/30 (D) 11/30

16. Given that the drawn ball from U2 is white, the probability that head appeared on the coin is
(A) 17/23 (B) 11/23 (C) 15/23 (D) 12/23

SECTION-IV (Total Marks : 28)
(Integer Answer Type)
This section contains 7 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9.
The bubble corresponding to the correct is to be darkened in the ORS.

17. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then find the value of $\frac{d}{d(\tan\theta)}\left(f(\theta)\right)$ Ans 1. Use trigonometry!!!!!

18. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is Ans: 2 Easy

19. The minimum value of the sum of real numbers a^{-5}, a^{-4}, 3 a^{-3}, 1, a^8 and a^{10} with a > 0 is Ans 8 Hint Aapply $wtA.M \geq wtG.M$

20. Let $f:[1,\infty) \to [2,\infty)$ be a differentiable function such that f(1)=2. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of f(2) is Ans: 6 Hint:

Differentiate $6\int_1^x f(t)dt = 3xf(x) - x^3$ to get $6f(x) = 3f(x) + 3xf'(x) - 3x^2$ which is a linear differential equation.

21. If z is any complex number satisfying |z – 3 – 2i| ≤ 2, then the minimum value of |2z – 6 + 5i| is

22. The positive integer value of n > 3 satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$ Ans: 7 Simple!! put $\frac{\pi}{n}=\theta$ for the hint!!

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