

Eliminate α from the equations

$$\frac{x}{a} \cos \alpha - \frac{y}{b} \sin \alpha = \cos 2\alpha$$

and

$$\frac{x}{a} \sin \alpha + \frac{y}{b} \cos \alpha = 2 \sin 2\alpha$$

Solution: Multiply the first equation by $\cos \alpha$ and second by $\sin \alpha$, then add

$$\begin{aligned} \cos \alpha \times \left(\frac{x}{a} \cos \alpha - \frac{y}{b} \sin \alpha \right) + \sin \alpha \times \left(\frac{x}{a} \sin \alpha + \frac{y}{b} \cos \alpha \right) \\ = \cos \alpha \cos 2\alpha + 2 \sin \alpha \sin 2\alpha \end{aligned}$$

$$\begin{aligned} \Rightarrow (\cos^2 \alpha + \sin^2 \alpha) \frac{x}{a} \\ = \cos \alpha (\cos 2\alpha + 4 \sin^2 \alpha) \quad (\text{since } \sin 2\alpha \\ = 2 \sin \alpha \cos \alpha) \end{aligned}$$

$$\Rightarrow \frac{x}{a} = \cos \alpha (1 + 2 \sin^2 \alpha) \dots (A) \quad (\text{since } \cos 2\alpha = 1 - 2 \sin^2 \alpha)$$

Now, multiply the first equation by $\sin \alpha$ and second by $\cos \alpha$, then subtract

$$\begin{aligned} \sin \alpha \times \left(\frac{x}{a} \cos \alpha - \frac{y}{b} \sin \alpha \right) - \cos \alpha \times \left(\frac{x}{a} \sin \alpha + \frac{y}{b} \cos \alpha \right) \\ = \sin \alpha \cos 2\alpha - 2 \cos \alpha \sin 2\alpha \end{aligned}$$

$$\Rightarrow -(\sin^2 \alpha + \cos^2 \alpha) \frac{y}{b} = \sin \alpha (\cos 2\alpha - 4 \cos^2 \alpha)$$

$$\Rightarrow \frac{y}{b} = -\sin \alpha (-2 \cos^2 \alpha - 1)$$

$$\Rightarrow \frac{y}{b} = \sin \alpha (2 \cos^2 \alpha + 1) \dots (B)$$

Now, add (A) with (B),

$$\begin{aligned}
& \frac{x}{a} + \frac{y}{b} = \cos \alpha (1 + 2 \sin^2 \alpha) + \sin \alpha (2 \cos^2 \alpha + 1) \\
\Rightarrow & \frac{x}{a} + \frac{y}{b} = \cos \alpha + \sin \alpha + 2 \sin \alpha \cos \alpha (\sin \alpha + \cos \alpha) \\
\Rightarrow & \frac{x}{a} + \frac{y}{b} = (\sin \alpha + \cos \alpha)(1 + 2 \sin \alpha \cos \alpha) \\
\Rightarrow & \frac{x}{a} + \frac{y}{b} = (\sin \alpha + \cos \alpha)(\sin^2 \alpha + \cos^2 \alpha + 2 \sin \alpha \cos \alpha) \\
\Rightarrow & \frac{x}{a} + \frac{y}{b} = \sin^3 \alpha + 3 \sin^2 \alpha \cos \alpha + 3 \sin \alpha \cos^2 \alpha + \cos^3 \alpha \\
\Rightarrow & \frac{x}{a} + \frac{y}{b} = (\sin \alpha + \cos \alpha)^3 \\
\sin \alpha + \cos \alpha &= \left(\frac{x}{a} + \frac{y}{b}\right)^{\frac{1}{3}} \dots \dots (C)
\end{aligned}$$

Similarly, subtracting (B) from (A) we get,

$$\cos \alpha - \sin \alpha = \left(\frac{x}{a} - \frac{y}{b}\right)^{\frac{1}{3}} \dots \dots (D)$$

Now, $(C)^2 + (D)^2$ gives

$$2 = \left(\frac{x}{a} + \frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{x}{a} - \frac{y}{b}\right)^{\frac{2}{3}}$$

which is the required equation.