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FIRST ORDER DERIVATIVE

1. If $y = f(\ln x)$ $find \frac{dy}{dx}$.

$$Solution: \frac{dy}{dx} = \frac{d}{dx}\left(f(\ln x)\right) = f'(\ln x) \times \frac{d}{dx} \ln x$$

$$f'(\ln x) \times \frac{1}{x} = \frac{f'(\ln x)}{x}$$

2. If $y = \ln\left(f(x)\right)$ $find \frac{dy}{dx}$.

$$Solution: \frac{dy}{dx} = \frac{d}{dx}\left(\ln\left(f(x)\right)\right) = \frac{1}{f(x)} \times \frac{d}{dx} f(x)$$

$$\frac{1}{f(x)} \times f'(x) = \frac{f'(x)}{f(x)}$$

3. If $y = ce^{\frac{x}{x-a}}$ $find \frac{dy}{dx}$.

$$Solution: \frac{dy}{dx} = \frac{d}{dx}\left(ce^{\frac{x}{x-a}}\right) = c \times \frac{d}{dx} e^{\frac{x}{x-a}}$$

$$= ce^{\frac{x}{x-a}} \times \frac{d}{dx}\left(\frac{x}{x-a}\right)$$

$$= ce^{\frac{x}{x-a}} \times \left(\frac{(x-a) \times \frac{d}{dx} x - x \times \frac{d}{dx}(x-a)}{(x-a)^2}\right)$$

$$= ce^{\frac{x}{x-a}} \times \left(\frac{(x-a) \times 1 - x \times 1}{(x-a)^2}\right)$$

$$= ce^{\frac{x}{x-a}} \times \left(\frac{(x-a) - x}{(x-a)^2}\right)$$

$$= -\frac{ace^{\frac{x}{x-a}}}{(x-a)^2} = -\frac{ay}{(x-a)^2}$$

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4. $If\ f(x) = 3e^{x^2}, find\ the\ value\ of\ f'(x) - 2xf(x) + \frac{1}{3}f(0) - f'(0).$

$$Solution:\quad f'(x) = \frac{d}{dx}3e^{x^2} = 3e^{x^2} \times \frac{d}{dx}x^2$$
$$= 3e^{x^2} \times 2x = 6xe^{x^2}$$

$$\therefore f'(0) = 0$$

$$now,\qquad 2xf(x) = 2x \times 3e^{x^2} = 6xe^{x^2}$$

$$also,\qquad \frac{1}{3}f(0) = \frac{1}{3}3e^0 = 1$$

$$\therefore f'(x) - 2xf(x) + \frac{1}{3}f(0) - f'(0) = 6xe^{x^2} - 6xe^{x^2} + 1 - 0 = 1$$

5. $If\ y = f(x)^{\varphi(x)}, find\ \frac{dy}{dx}$

$$y = f(x)^{\varphi(x)}$$

$$\Rightarrow \ln y = \ln f(x)^{\varphi(x)} = \varphi(x)\ln f(x)$$

$$now\ differentiating\ both\ side\ w.r.t\ 'x'$$
$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\varphi(x)\ln f(x))$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \varphi(x)\frac{d}{dx}\ln f(x) + \ln f(x)\frac{d}{dx}\varphi(x)$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\varphi(x)f'(x)}{f(x)} + \varphi'(x)\ln f(x)\right\}$$

$$\Rightarrow \frac{dy}{dx} = f(x)^{\varphi(x)}\left\{\frac{\varphi(x)f'(x)}{f(x)} + \varphi'(x)\ln f(x)\right\}$$

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6. $If\ f(x) = x + 1, find \frac{d}{dx}\big(\ fof(x)\big)$

$$fof(x) = f\big(f(x)\big) = f(x+1) = (x+1) + 1 = x + 2$$

$$\therefore \frac{d}{dx}\big(\ fof(x)\big) = \frac{d}{dx}(x+2) = 1$$

7. Find the derivatives of the following functions
   i. $y = (x+15)^9$
   ii. $y = \frac{1}{\sqrt{ax+b}}$
   iii. $y = \sqrt{ax^2 + bx + c}$
   iv. $y = \frac{1}{\sqrt{ax^2+bx+c}}$
   v. $y = \sqrt{x} - \frac{1}{\sqrt{x}}, x > 0$
   vi. $y = \sqrt{\frac{1+x}{1-x}}$
   vii. $y = \sin x^o$
   viii. $y = \tan(ax+b)$
   ix. $y = \tan\{f(x)\}$
   x. $y = \sin\{f(x)\}^2$
   xi. $y = \frac{\cos x}{1+\sin x}$
   xii. $y = \frac{\cos x - \cos 2x}{1-\cos x}$

Solution i.

$$\frac{dy}{dx} = \frac{d}{dx}(x+15)^9$$

$$\Rightarrow \frac{dy}{dx} = 9(x+15)^8 \times \frac{d}{dx}(x+15)$$

$$\Rightarrow \frac{dy}{dx} = 9(x+15)^8$$

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Solution ii.

$$\frac{dy}{dx} = \frac{d}{dx}\frac{1}{\sqrt{ax+b}} = \frac{d}{dx}(ax+b)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}(ax+b)^{-\frac{1}{2}-1} \times \frac{d}{dx}(ax+b)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}(ax+b)^{-\frac{3}{2}} \times a$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}a\frac{1}{(ax+b)^{\frac{3}{2}}} = -\frac{a}{2(ax+b)\sqrt{ax+b}}$$

Solution iii.

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{ax^2+bx+c} = \frac{d}{dx}(ax^2+bx+c)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(ax^2+bx+c)^{\frac{1}{2}-1} \times \frac{d}{dx}(ax^2+bx+c)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(ax^2+bx+c)^{-\frac{1}{2}} \times (2ax+b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\frac{2ax+b}{(ax^2+bx+c)^{\frac{1}{2}}} = -\frac{2ax+b}{2\sqrt{ax^2+bx+c}}$$

Solution v.

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{x} - \frac{d}{dx}\frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}} = \frac{1}{2\sqrt{x}}\left(1 + \frac{1}{x}\right)$$

$$\boldsymbol{Important\ Formula:\ \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}}$$

$$\boldsymbol{and\ \ \frac{d}{dx}\frac{1}{\sqrt{x}} = -\frac{1}{2x\sqrt{x}}}$$

Solution vi.

Put $\frac{1+x}{1-x} = z$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\sqrt{z} = \frac{1}{2\sqrt{z}}\frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1+x}{1-x}}}\frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1-x}{1+x}} \times \left\{\frac{(1-x) \times \frac{d}{dx}(1+x) - (1+x) \times \frac{d}{dx}(1-x)}{(1-x)^2}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1-x}{1+x}} \times \left\{\frac{(1-x) \times 1 - (1+x) \times (-1)}{(1-x)^2}\right\}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1-x}{1+x}} \times \left\{\frac{1-x+1+x}{(1-x)^2}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1-x}{1+x}} \times \left\{\frac{2}{(1-x)^2}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-x}{1+x}} \times \left\{\frac{1}{(1-x)^2}\right\} = \frac{1}{(1-x)(1-x)}\sqrt{\frac{1-x}{1+x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1-x)\sqrt{1-x}}\frac{1}{\sqrt{1+x}} = \frac{1}{(1-x)\sqrt{1-x^2}}$$

$$\boldsymbol{Important\ Differentials}: \frac{d}{dx}\left(\frac{1+x}{1-x}\right) = \frac{2}{(1-x)^2}$$

$$\boldsymbol{Similarly}\ \frac{d}{dx}\left(\frac{1-x}{1+x}\right) = \frac{-2}{(1+x)^2}$$

$$\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{-4x}{(1+x^2)^2}$$

$$\frac{d}{dx}\left(\frac{1+x^2}{1-x^2}\right) = \frac{4x}{(1-x^2)^2}$$

You can prove these easily by using the division rule.

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Solution vii.

$$y = \sin x^o$$

$$\Rightarrow y = \sin\frac{\pi x}{180} \ (\textit{convert to radians})$$

$$\Rightarrow \frac{dy}{dx} = \cos\frac{\pi x}{180} \times \frac{d}{dx}\frac{\pi x}{180}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\pi}{180}\cos\frac{\pi x}{180}$$

Solution ix/ viii.

$$y = \tan f(x)$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 f(x) \times \frac{d}{dx} f(x)$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 f(x) \times f'(x) = f'(x)\sec^2 f(x)$$

For problem viii. put $f(x) = ax + b$ *and see that* $f'(x) = a$

Solution x.

$$y = \sin\{f(x)\}^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\sin\{f(x)\}^2$$

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$$\Rightarrow \frac{dy}{dx} = \cos\{f(x)\}^2 \times \frac{d}{dx} f(x)^2$$

$$\Rightarrow \frac{dy}{dx} = \cos\{f(x)\}^2 \times 2f(x) \frac{d}{dx} f(x)$$

$$\Rightarrow \frac{dy}{dx} = \cos\{f(x)\}^2 \times 2f(x) \times f'(x)$$

Solution xi.

$$y = \frac{\cos x}{1 + \sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \frac{\cos x}{1 + \sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \sin x) \times \frac{d}{dx} \cos x - \cos x \ \times \frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \sin x) \times (-\sin x) - \cos x \ \times (\cos x)}{(1 + \sin x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1 + \sin x)^2} = -\frac{\sin x}{(1 + \sin x)^2}$$

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Solution xii.

$$y = \frac{\cos x - \cos 2x}{1 - \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos x - \cos 2x}{1 - \cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 - \cos x) \times \frac{d}{dx}(\cos x - \cos 2x) - (\cos x - \cos 2x) \times \frac{d}{dx}(1 - \cos x)}{(1 - \cos x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 - \cos x) \times (-\sin x + 2\sin 2x) - (\cos x - \cos 2x) \times (\sin x)}{(1 - \cos x)^2}$$

$$= \frac{-\sin x + \sin x \cos x + 2\sin 2x - 2\cos x \sin 2x - \sin x \cos x + \sin x \cos 2x}{(1 - \cos x)^2}$$

$$= \frac{-\sin x + 4\sin x \cos x - 4\cos^2 x \sin x + \sin x \cos 2x}{(1 - \cos x)^2}$$

$$= \frac{\sin x\,(-1 + 4\cos x - 4\cos^2 x + \cos 2x)}{(1 - \cos x)^2}$$

$$= \frac{\sin x\,(-1 + 4\cos x - 4\cos^2 x + 2\cos^2 x - 1)}{(1 - \cos x)^2}$$

$$= \frac{\sin x\,(-2\cos^2 x + 4\cos x - 2)}{(1 - \cos x)^2}$$

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$$= \frac{-2\sin x\,(\cos^2 x - 2\cos x + 1)}{(1-\cos x)^2}$$

$$= \frac{-2\sin x\,(1-\cos x)^2}{(1-\cos x)^2}$$

$$= -2\sin x$$

8. Find the derivatives of the following functions
   i. $y = e^{9x}$
   ii. $y = e^{2x^3+3x^2+4x+5}$
   iii. $y = e^{\sin^2 x}$
   iv. $y = 5^{x\log_5 7}$

$Solution\ i - iii.$

$$if\ y = e^{f(x)}$$

$$\Rightarrow \frac{dy}{dx} = e^{f(x)} \times f'(x)$$

$$i. \qquad f(x) = 9x, therefore\ f'(x) = 9$$

$$\Rightarrow \frac{dy}{dx} = e^{9x} \times 9 = 9e^{9x}$$

$$ii. \qquad f(x) = 2x^3 + 3x^2 + 4x + 5, therefore$$
$$f'(x) = 6x^2 + 6x + 4$$

$$\Rightarrow \frac{dy}{dx} = e^{2x^3+3x^2+4x+5} \times (6x^2 + 6x + 4)$$
$$= (6x^2 + 6x + 4)e^{2x^3+3x^2+4x+5}$$

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$iii.$ $\quad f(x) = \sin^2 x\,, therefore\ f'(x) = 2\sin x \cos x$

$$\Rightarrow \frac{dy}{dx} = e^{\sin^2 x} \times 2\sin x \cos x = \sin 2x\, e^{\sin^2 x}$$

$Solution\ iv.$

$$y = 5^{x log_5 7}$$

$$\Rightarrow y = 5^{\log_5 7^x}$$

$$\Rightarrow y = 7^x$$

$$\ln y = \ln 7^x = x \ln 7$$

Now differentiate both side w.r.t 'x'

$$\frac{1}{y}\frac{dy}{dx} = \ln 7$$

$$\frac{dy}{dx} = y \ln 7 = 7^x \ln 7$$

9. Find the derivatives of the following functions
   i. $y = \ln(ax^2 + bx + c)$
   ii. $y = \ln(\tan(2x + 3))$
   iii. $y = \sqrt{\ln(ax + b)}$
   iv. $y = \ln(\ln x)$
   v. $y = \ln(\sin(\log x))$
   vi. $y = \ln(\sec x + \tan x)$

Using chain rule see that if

$$\boldsymbol{y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)}$$

$Solution\ i.$

$$y = \ln(ax^2 + bx + c)$$

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$$\Rightarrow \frac{dy}{dx} = \frac{1}{ax^2 + bx + c} \times \frac{d}{dx}(\, ax^2 + bx + c)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2ax + b}{ax^2 + bx + c}$$

Solution ii.

$$y = \ln(\tan(2x + 3))$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\tan(2x + 3)} \times \frac{d}{dx}(\, \tan(2x + 3))$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2(2x + 3)}{\tan(2x + 3)} \times \frac{d}{dx}(\, 2x + 3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sec^2(2x + 3)}{\tan(2x + 3)}$$

Solution iii.

$$y = \sqrt{\ln(ax + b)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\ln(ax + b)}} \times \frac{d}{dx}(\, \ln(ax + b))$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\ln(ax + b)}} \times \frac{1}{ax + b}\frac{d}{dx}(ax + b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{a}{2(ax + b)\sqrt{\ln(ax + b)}}$$

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Solution iv.

$$y = \ln(\ln x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\ln x} \times \frac{d}{dx}(\ln x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x \ln x}$$

Solution v.

$$y = \ln(\sin(\ln x))$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sin(\ln x)} \times \frac{d}{dx}(\sin(\ln x))$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(\ln x)}{\sin(\ln x)} \times \frac{d}{dx}(\ln x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cot(\ln x)}{x}$$

Solution vi.

$$y = \ln(\sec x + \tan x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sec x + \tan x} \times \frac{d}{dx}(\sec x + \tan x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec x\,(\tan x + \sec x)}{\sec x + \tan x}$$

$$\Rightarrow \frac{dy}{dx} = \sec x$$

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10. From the given parametric equations, find the differential coefficient.

i. $x = a\cos t\ , y = b\sin t$

ii. $x = at^2\ , y = 2at$

iii. $x = a(t - \sin t)\ , y = a(1 - \cos t)$

iv. $x = t^2 + 2t\ , y = t^3 - 3t$

General approach for parametric equations:

$$if\ x = \varphi(t)\ and\ y = \omega(t)$$

$$\Rightarrow \frac{dx}{dt} = \varphi'(t)\ and\ \frac{dy}{dt} = \omega'(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\omega\prime(t)}{\varphi'(t)}$$

Solution i.

$$\frac{dy}{dx} = \frac{b\cos t}{-a\sin t} = -\frac{b}{a}\cot t$$

Solution ii.

$$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

Solution iii.

$$\frac{dy}{dx} = \frac{a\sin t}{a(1-\cos t)} = \frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{2\sin^2\frac{t}{2}} = \cot\frac{t}{2}$$

Solution iv.

$$\frac{dy}{dx} = \frac{3t^2 - 3}{2t + 2} = \frac{3(t-1)(t+1)}{2(t+1)} = \frac{3}{2}(t-1)$$

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11. Find the differential coefficients of the following functions

i. $y^2 - 4ax = 0$

ii. $x^2 + y^2 = a^2$

iii. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

iv. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

v. $ax^2 + 2hxy + by^2 = 1$

vi. $\log_e(xy) = x^2 + y^2$

$$For\ functions\ of\ the\ form\ f(x,y) = c, where$$

$$y\ can\ or\ cannot\ be\ explicitly\ written\ in\ terms\ of\ x,$$

$$to\ find\ \frac{dy}{dx}\ differentiate\ both\ side\ w.r.t\ 'x'$$

$$and\ then\ make\ \frac{dy}{dx}\ the\ subject.$$

Solution i.

$$\frac{d}{dx}(y^2 - 4ax) = \frac{d}{dx}(0)$$

$$\Rightarrow 2y\frac{dy}{dx} - 4a = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$$

Solution ii.

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(a^2)$$

$$\Rightarrow 2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

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Solution iii.

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

Solution iv.

$$\frac{d}{dx}\left(x^{\frac{2}{3}}+y^{\frac{2}{3}}\right)=\frac{d}{dx}(a^{\frac{2}{3}})$$

$$\Rightarrow \frac{2}{3}x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}}=-\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Solution v.

$$\frac{d}{dx}(ax^2+2hxy+by^2)=\frac{d}{dx}(1)$$

$$\Rightarrow 2ax+2h\frac{d}{dx}(xy)+2by\frac{dy}{dx}=0$$

$$\Rightarrow ax+h\left(x\frac{dy}{dx}+y\right)+by\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}(hx+by)=-(ax+hy)$$

$$\Rightarrow \frac{dy}{dx}=-\frac{(ax+hy)}{(hx+by)}$$

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Solution vi.

$$\frac{d}{dx}(\ln xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\Rightarrow \frac{1}{xy}\frac{d}{dx}(xy) = 2x + 2y\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{xy}\left(x\frac{dy}{dx} + y\right) = 2x + 2y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - 2y\right) = 2x - \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - 2y^2}{y}\right) = \frac{2x^2 - 1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(2x^2 - 1)}{x(1 - 2y^2)}$$

12. $If\ f(x) = \ln x\,, find\ the\ derivative\ of\ f(\ln x\ ).$

Solution:

$$f(\ln x) = \ln(\ln x)$$

$$\Rightarrow \frac{d}{dx}\ f(\ln x) = \frac{d}{dx}\ln\ln x$$

$$\Rightarrow \frac{f'(\ln x)}{x} = \frac{1}{x\ln x}$$

$$\Rightarrow f'(\ln x) = \frac{1}{\ln x}$$

13. $If\ y = \ln\left(\tan\frac{x}{2}\right), find\ \frac{dy}{dx}$

Solution:

$$y = \ln\left(\tan\frac{x}{2}\right)$$

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$$\Rightarrow \frac{dy}{dx} = \frac{1}{\tan\frac{x}{2}} \frac{d}{dx}\left(\tan\frac{x}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}} = \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{1}{\sin x} = cosec\ x$$

14. $If\ y = \ln(x + \sqrt{x^2 \pm a^2})\,, find\ \frac{dy}{dx}$

$$y = \ln(x + \sqrt{x^2 \pm a^2})$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 \pm a^2}} \frac{d}{dx}\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 \pm a^2}}\left(1 + \frac{1}{2\sqrt{x^2 \pm a^2}} \frac{d}{dx}(\,x^2 \pm a^2)\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 \pm a^2}}\left(1 + \frac{2x}{2\sqrt{x^2 \pm a^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 \pm a^2}}\left(\frac{x + \sqrt{x^2 \pm a^2}}{\sqrt{x^2 \pm a^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2 \pm a^2}}$$

15. $If\ y = \log_x e\,, then\ show\ that\ x\frac{dy}{dx} + y^2 = 0.$

Solution:

$$y = \log_x e = \frac{1}{\log_e x} = (\log_e x)^{-1}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{(\log_e x)^2} \times \frac{d}{dx}(\log_e x)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x(\log_e x)^2}$$

$$\Rightarrow x\frac{dy}{dx} = -(\log_x e)^2 = -y^2$$

$$\Rightarrow x\frac{dy}{dx} + y^2 = 0$$

16. $if\ x^y = e^x\ then\ find\ \frac{dy}{dx}$

Solution:

$$x^y = e^x$$

$$\Rightarrow \ln x^y = \ln e^x$$

$$\Rightarrow y\ln x = x\ln e = x$$

$$\Rightarrow \frac{d}{dx}(y\ln x) = \frac{d}{dx}x$$

$$\Rightarrow \frac{y}{x} + \ln x\frac{dy}{dx} = 1$$

$$\Rightarrow \ln x\frac{dy}{dx} = \frac{x-y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x\ln x}$$

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