Differential Equation

1)Find the order and degree of the following differential equations
i)$(\frac{dy}{dx})^2 - 2\frac{dy}{dx} = 3x$
ii)$\frac{d^3y}{dx^3} + (\frac{dy}{dx})^2 + 4y = 0$
iii)$\frac{d^2y}{dx^2} = [1 + (\frac{dy}{dx})^2]^{\frac{2}{3}}$
2) Find the differential equation by eliminating the arbitrary constants a,b,A and B
i) $y = a\log x + b$
ii)$y = Ae^x + Be^{-x}$
iii)$y = Ae^x + Be^{-x} + x^2$
Hint: You can differetiate the functions atmost the number of constant in the function
Answer i) $xy_2 + y_1 = 0$ ii)$y_2 - y = 0$ iii)$y_2 - y + x^2 - 2 = 0$

3)Show that the differential equation whose general solution is $y = a\sin(bx + c)$ where a,b,c are constants is $yy_3 = y_1y_2$

Hint: On $diff^n$ we get $y_1 = a\cos(bx + c).b$ again differentiate it to get $y_2 = -a\sin(bx + c).b^2$ which is equal to $y_2 = -yb^2$ now differentiating it we get $y_3 = -y_1b^2$ now equate the value of b^2 to get the answer

4) Show that the equation of the curve passing through the point $(2, 1)$ and having the gradient $-y$ at any point (x, y) on it is $\log|y| = 2 - x$

Hint: You have to solve the differential equation $\frac{dy}{dx} = -y$

5) A curve passes through the point $(4, 3)$ and the slope of the tangent to the curve at any point is equal to the reciprocal of the ordinate at that point, then show that the equation of the curve is $y^2 = 2x + 1$

Hint: You have to solve the differential equation $\frac{dy}{dx} = \frac{1}{y}$

6) Solve $3e^x \tan y dx + (1 - e^x)\sec^2 y dy$

Hint: Answer $\tan^2 y = c^2(1 - e^x)^6$

7) Solve $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$, given y=1 when x=1.

Hint: $\frac{xdy - ydx}{x^2 + y^2} = \frac{xdy - ydx}{x^2}\frac{1}{1 + (\frac{y}{x})^2} = d(\frac{y}{x})\frac{1}{1 + (\frac{y}{x})^2}$ now proceed

8) Solve $x\frac{dy}{dx} = y(\log y - \log x + 1)$

Hint: Answer $\log(\frac{y}{x}) = cx$ the given equation can be written as $\frac{d(\frac{y}{x})}{\log(\frac{y}{x})\frac{y}{x}} = \frac{dx}{x}$

Vinod Kumar Singh
9874059952
http://sites.google.com/site/mathsvinu