Definite Integral

1. Show that $\int_1^3 [x^2]dx = 5-\sqrt{3}-\sqrt{2}$ Hint: Consider the intervals $(1,\sqrt{2}),(\sqrt{2},\sqrt{3}),(\sqrt{3},2)$.
2. Show that $\int_{\pi/4}^{3\pi/4} \frac{\varphi d\varphi}{1+\sin\varphi} = (\sqrt{2}-1)\pi$ Hint: Use $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
3. Show that $\int_{-2}^{2} |1-x^2|dx = 4$ Hint: $|1-x^2|$ is an even function so the given integral is $2\int_0^2 |1-x^2|dx$, now consider the interval $(0,1),(1,2)$.
4. Show that $\int_0^{\pi/4} \frac{(\sin x+\cos x)dx}{9+16\sin 2x} = \frac{1}{20}\log 3$ Hint: $9+16\sin 2x = 25-16(\sin x-\cos x)^2$
5. Show that $\int_0^{\pi/4} \log(1+\tan x)dx = \frac{\pi}{8}\log 2$ Hint: $\log\left(1+\tan(\frac{\pi}{4}-x)\right) = \log\left(1+\frac{1-\tan x}{1+\tan x}\right) = \log 2 - \log(1+\tan x)$
6. Evaluate: $\int_{-2}^{2} (|x|+|x-1|)\,dx$
7. Show that $\int_0^{2011} e^{x-[x]}dx = 2011(e-1)$: Hint $\int_0^{2011} e^{x-[x]}dx = \sum_{k=0}^{2010}\int_k^{k+1} e^{x-[x]}dx = \sum_{k=0}^{2010}\int_k^{k+1} e^{x-k}\,dx$
8. Show that $\int_0^{\pi} |\sin x+\cos x|dx = 2\sqrt{2}$ Hint: $\sin x+\cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$, it follows that $\sin\left(x+\frac{\pi}{4}\right) > 0, 0 < x < \frac{3\pi}{4}$ and $\sin\left(x+\frac{\pi}{4}\right) < 0, \frac{3\pi}{4} < x < \pi$
9. Show that $\int_0^{\pi} \frac{xdx}{(a^2\cos^2 x+b^2\sin^2 x)^2} = \frac{\pi^2(a^2+b^2)}{4a^2b^3}$ Hint: Apply the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and observe that $\frac{1}{(a^2\cos^2 x+b^2\sin^2 x)^2} = \frac{\sec^4 x}{(a^2+b^2\tan^2 x)^2} = \frac{(1+\tan^2 x)\sec^2 x}{(a^2+b^2\tan^2 x)^2}$ and use the substitution $b\tan x = a\tan\theta$

10. Show that $\int_0^{\pi/2} \log(\sin x)dx = \frac{\pi}{2}\log\frac{1}{2}$ Hint: Let I be the given integral, then $I = \int_0^{\pi/2} \log(\cos x)dx$ now

$$2I = \int_0^{\pi/2} \log(\sin x \cos x)dx = \int_0^{\pi/2} \log(\frac{\sin 2x}{2})dx = \int_0^{\pi/2} \log(\sin 2x)dx - \int_0^{\pi/2} \log 2dx$$

11. Evaluate $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$ Hint: Put $x = \tan\varphi$ so

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int_{-\pi/6}^{\pi/6} \frac{\tan^4\varphi}{1-\tan^4\varphi}\left(\frac{\pi}{2}-2\varphi\right)\sec^2\varphi d\varphi$$ Now factories the denominator and think of odd and even functions!

12. Show that $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx$ Hint:

$$\int_0^{\pi/2} f(\sin 2x)\sin x dx = \int_0^{\pi/4} f(\sin 2x)\sin x dx + \int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x dx$$ and Put

$x = \frac{\pi}{2} - \varphi$ in $\int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x dx$ so that

$$\int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x dx = \int_0^{\pi/4} f(\sin 2\varphi)\sin\varphi d\varphi$$, now proceed!

13. Show that $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2} = \pi\log 2$ Hint: put $x = \tan\theta$

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