

Find  $y = \frac{x}{1+x}$  given  $x = \omega^{2009^{2009 \dots 2009 \text{ times}}}$

First we note that  $\omega^{2009} = (\omega^3)^{669} \cdot \omega^2 = \omega^2 \dots \dots \dots (A)$  and

$(\omega^2)^{2009} = \omega^{4018} = (\omega^3)^{1339} \cdot \omega = \omega \dots \dots \dots (B)$

Now

$$\begin{aligned}
 x &= \omega^{2009^{2009 \dots 2009 \text{ times}}} = (\omega^2)^{2009^{2009 \dots 2008 \text{ times}}} \\
 &= \omega^{2009^{2009 \dots 2007 \text{ times}}}
 \end{aligned}$$

(In the above calculation we have done two power operation i.e, one pair of power operation) Continuing like these after every pair of power operation we get back

**$\omega$**

Pairing the powers (which are 2009 times) we will have to perform 1008+1 power operation in which the first 1008 operations will give back  **$\omega$**  by (A) and (B)

Eventually  $x = \omega^{2009} = \omega^2$

Therefore  $y = \frac{\omega^2}{1+\omega^2} = \frac{\omega^2}{-\omega} = -\omega$