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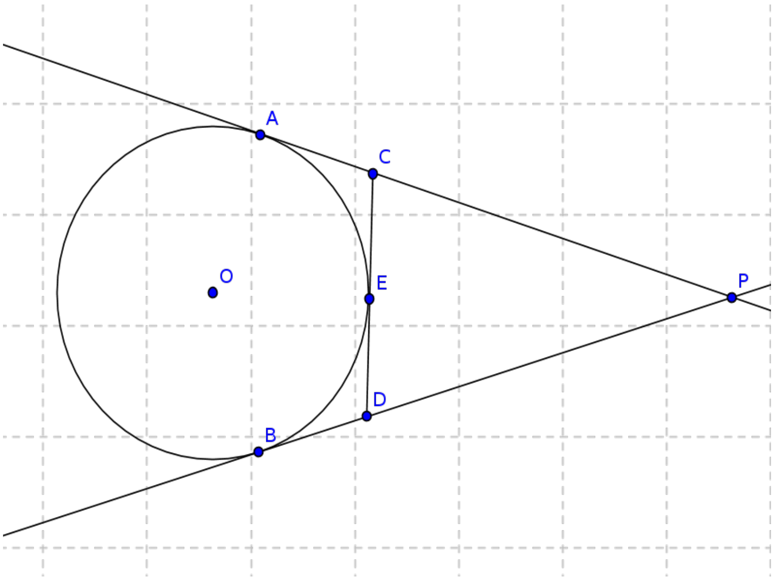
CIRCLES

1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm , find the perimeter of ΔPCD.



Let AC = x cm. Then PC = AP – AC = 14−x.

AP = BP (Tangents from the same external points are equal in length)

Simlarly if BD = y cm, then PD = 14−y

Again AC = CE = x cm and ED = DB = y cm (Tangents from the same.........)

Perimeter of **Δ**PCD = PC+CD+DP

= 14−x+CE+ED+14−t

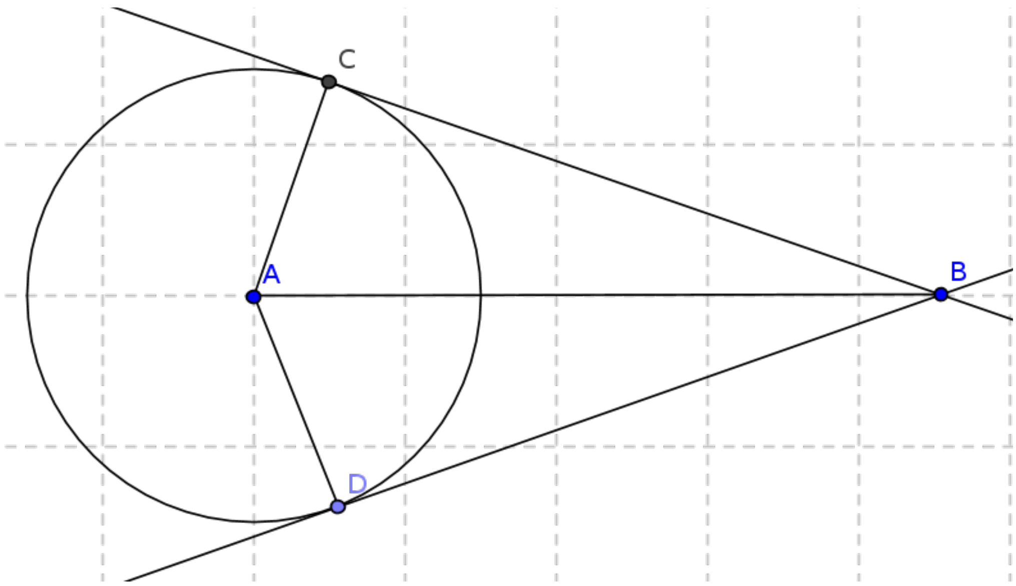
= 14−x+x+y+14−y

=28 cm

2. AP and BP are tangents from an external point P to a circle with centre O. CD is another tangent which touches the circle at E. If AP = 11 cm and ED = 4cm, find the length of DP.

Same as above. (Refer to the diagram in problem 1)

3. In the figure given $\angle ABC = 30^{\circ}$. Find ∠BAC and ∠**CAD** where BC and BD are tangents to the circle.



Since BC is the tangent and A is the centre of the

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circle. AC is perpendicular to BC. Thus in ΔABC

∠CAB+∠ACB+∠ABC = 180°

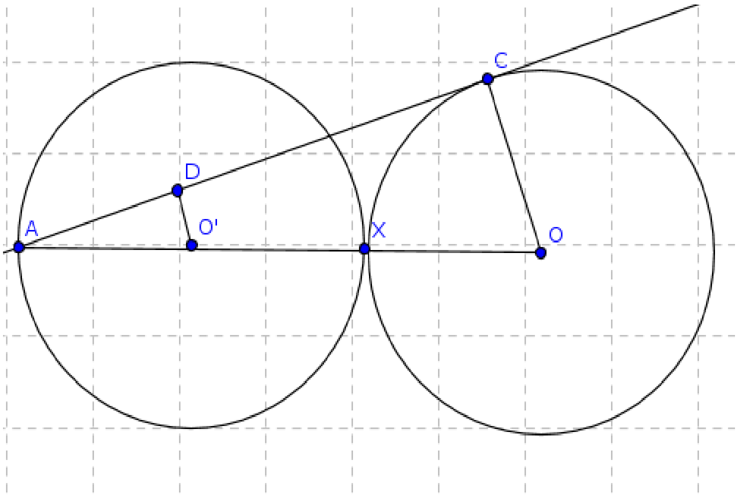
=> ∠CAB = 180 – 90 – 30 = 60°

Also the ΔABC & ΔABC are congruent (determine yourself)

∠CAB = ∠DAB

Therefore ∠CAD = 120°

4. Equal circles with centre O and O' touch each other at X. OO' is produced to meet a circle with centre O', at A. AC is a tangent to the circle whose centre is O. O'D is perpendicular to AC. Find the value of DO'/CO.



Consider the ΔAO'D and ΔAOC

∠O'AD = ∠OAC

∠O'DA = 90° (given)

∠OCA = 90° (Since AC is a tangent and the line joining the centre and the point of contact are perpendicular)

So, ΔAO'D ~ ΔAOC

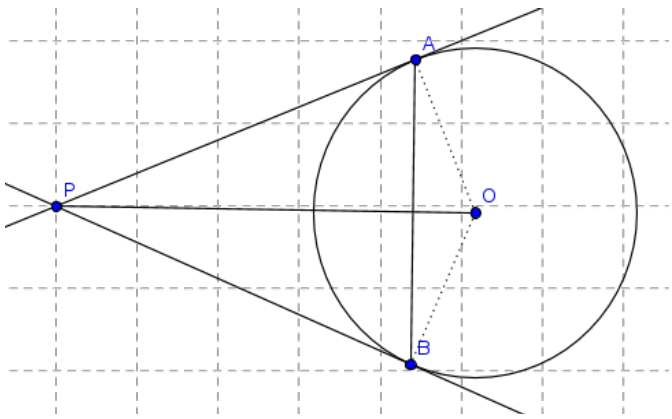
Therefore

$$\frac{DO'}{CO} = \frac{AO'}{AO}$$

$$\Rightarrow \frac{DO'}{CO} = \frac{AO'}{3 \times AO'} = \frac{1}{3}$$

Since the radius of the two circles are equal AO = 3AO'

5. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP is equal to the diameter of the circle , show that PO bisects AB and ΔPAB is equilateral.



Constructions: Join the points A and O, B and O.

Now consider the ΔPOA and ΔPOB

∠OAP = ∠OBP = 90° (line joining the centre and the point of contact are perpendicular)

OA = OB (radius of the same circle)

PA = PB (length of tangents from the external point are equal)

Therefore $\nabla POA \simeq \nabla POB$

∠BPO = ∠APO = y (say)

Let PO cuts AB at X.

∠OBA = ∠OAB = z, say(base of an isosceles triangle)

∠PBX = ∠PBO – ∠OBA = 90 – z

∠PAX = ∠PAO = ∠OAB = 90 – z

Therefore ∠PBX = ∠PAX

consider the ΔPXA and ΔPXB

Since ∠BPO = ∠APO and ∠PBX = ∠PAX already proved

so ∠PXB = ∠PXA = 90 (since they are on a st. line)

also note that PA = PB and PX is common, so ΔPXA congruent ΔPXB

SO AX = XB (c.p.c.t)

This proves that PX i.e., PO bisects AB

Let the radius of the circle be r.

Given OP = 2r

In ΔPOA by Pythagoras theorem

$$PA^2 = OP^2 - OA^2$$
$$\Rightarrow PA^2 = 4r^2 - r^2 = 3r^2$$
$$\Rightarrow PA = \sqrt{3}r$$
$$So\ PA = PB = \sqrt{3}r$$
$$\tan(\angle AOP) = \frac{PA}{OA} = \frac{\sqrt{3}r}{r} = \sqrt{3}$$
$$\Rightarrow \angle AOP = 60$$
$$\tan(\angle AP0) = \frac{OA}{PA} = \frac{r}{\sqrt{3}r} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \angle APO = 30$$

Similarly ∠BPO = 30, again ∠PAB = ∠PBA since (PA = PB)

so in ΔPBA

2∠PAB + 60 = 180

=> ∠PAB = 60

=> ∠PBA = 60

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∠APB = ∠APO +∠BPO = 30 + 30 = 60

This is what was required! I have deduced few extra conditions which is not required in the problem but might be useful in some other application or deduction.

6. Two tangents PA and PB are drawn to a circle with centre O such that ∠BPA = 120 degrees. Prove that OP = 2PA

From the above discussion in problem 5

$$\nabla POA \simeq \nabla POB$$

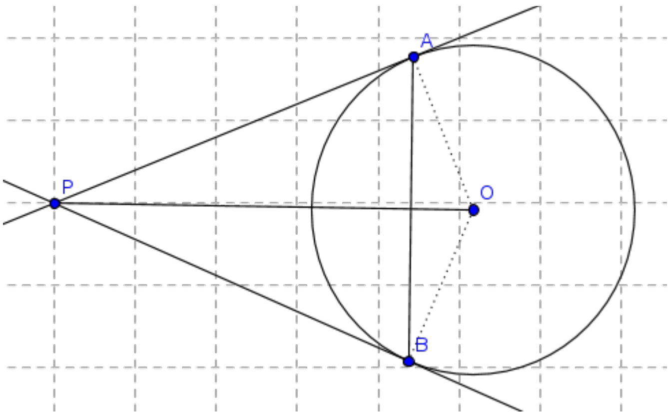
So ∠BPO = ∠APO = 60 degrees

In ΔPAO

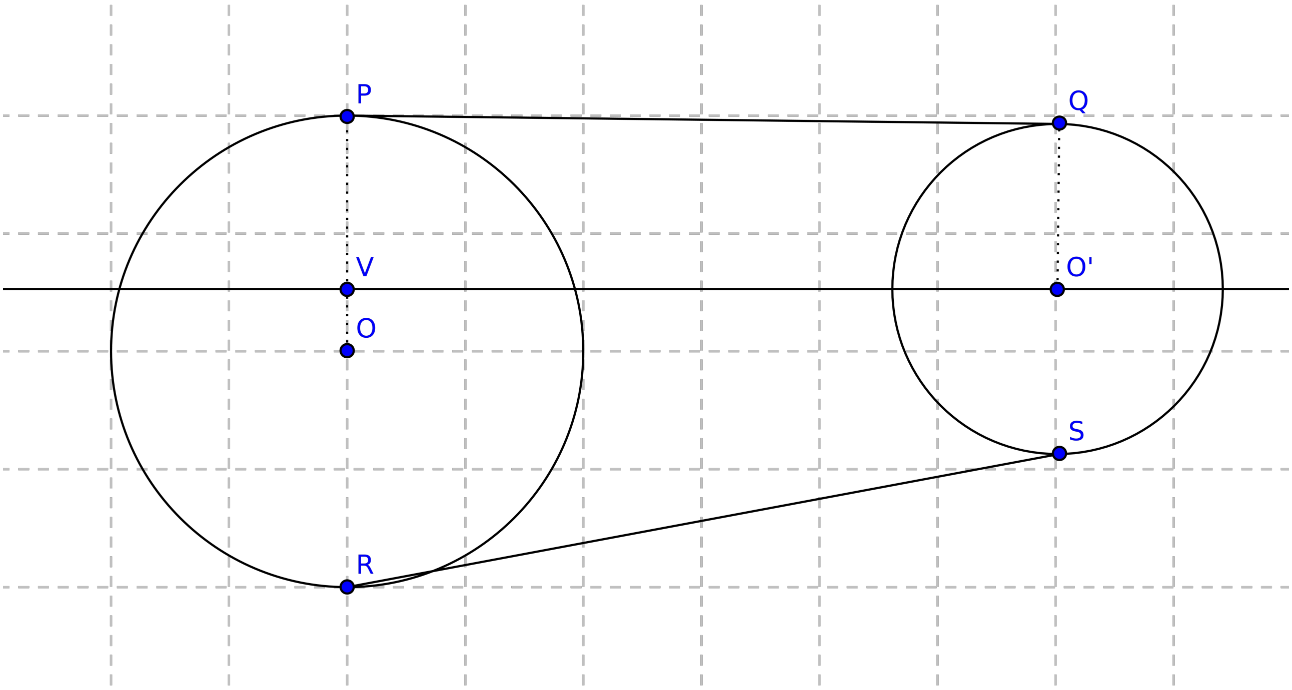
$$\cos(60^o) = \frac{PA}{OP}$$

$$\Rightarrow \frac{1}{2} = \frac{PA}{OP}$$

$$\Rightarrow OP = 2PA$$



7.PQ and RS are commont tangents to two circles of unequal radii. Prove that PQ = RS.



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Do it yourself!