B.Math. (Hons.) Admission Test: 2007

Short-Answer Type Test

Time: 2 hours

1. Let n be a positive integer. If n has odd number of divisors (other than 1 and n), then show that n is a perfect square.
2. Let a and b be two non-zero rational numbers such that the equation $ax^2 + by^2 = 0$ has no non-zero solution in rational numbers. Prove that for any rational number t, there is a rational solution of the equation $ax^2 + by^2 = t$.
3. For any natural number $n > 1$, consider the n-1 points on the unit circle $e^{2\pi ik/n}(k = 1, 2, ..., n-1)$. Show that the sum of the distances of these points from 1 is n.
4. Let ABC be an isosceles triangle with AB=AC=20. Let P be a point inside the triangle ABC such that the sum of the distances of P to AB and AC is 1. Describe the locus of all such points inside ABC.
5. Let $P(X)$ be a polynomial with integer coefficients of degree d >0.
   (a) If α and β are two integers such that $P(\alpha)=1$ and $P(\beta) = -1$, then prove that $|\beta - \alpha|$ divides 2.
   (b) Prove that the number of distinct integer roots of $P^2(x) - 1$ is at most d+2.
6. In ISI club each member is on two committees and any two committees have exactly one member in common. There are five committees. How many members does ISI Club have?
7. Let $0 \le \theta \le \frac{\pi}{2}$. Prove that $\sin\theta \ge \frac{2\theta}{\pi}$
8. Let $P: ¡ \to ¡$ be a continuous function such that $P(X) = X$ has no real solution. Prove that $P(P(X)) = X$ has no real solution.
9. In a group of five people any two are either friends or enemies, no three of them are friends of each other and no three of them are enemies of each other. Prove that every person in this group has exactly two friends.
10. The eleven members of a cricket team are numbered 1,2,3,…….,11. In how many ways can the entire cricket team sit on the eleven chairs arranged a circular table so that the numbers of any two adjacent players differ by one or two?

Best of luck!

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